University of North Georgia
Department of Mathematics

Instructor: Berhanu Kidane
Course: College Algebra Math 1111
Text Book: For this course we use the free e – book by Stitz and Zeager with link:
http://www.stitz-zeager.com/szca07042013.pdf
Other online resources:
e – book: http://msenux.redwoods.edu/IntAlgText/
Tutorials: http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/index.htm

For more free supportive educational resources consult the syllabus

Chapter 1

Review on Sets and the Real Number systems

See review notes on sets and the real numbers

Review on Equations and Inequalities

Objectives: by the end of these sections students should be able to:

- **Identify and solve:**
  - linear Equations
  - Quadratic Equations
  - Polynomial Equations
  - Equations Involving Radicals
  - Equations of Quadratic Type
- **Identify and Solve:**
  - Linear Inequalities
  - Non-linear Inequalities (Quadratic Inequalities, Rational inequalities)
  - Some Application Problems
- **Identify and solve:**
  - Absolute Value Equations and Inequalities

In college algebra it is assumed that students have the mastery of basic algebraic properties, but students are strongly encouraged to review the following online resources:

OER from West Texas A&M University **Tutorials 2 -12**

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/index.htm

Equations and Inequalities

In this section students learn how to solve equations and inequalities

Equations

Solving Equations: Solving equations involve one or more of the following principles

- **Add or Subtract the same value from both sides**
- **Divide every term by the same nonzero value**
- **Clear out any fractions by Multiplying every term by the least common denominator (LCD)**
- **Combine Like Terms**
- **Factoring or Expanding**
- **Cancelling common factors from the numerator and Denominator**

1) Linear equations

Example: Solve $2x - \frac{3}{4} = \frac{1}{2}x + 9$

OER (Open Educational Resources) from West Texas A&M, **Tutorial 14**: Linear Equation in One Variable

2) Equations Involving Fractions

Example: Solve $\frac{1}{2}x - \left(x - \frac{1}{3}\right) = -\frac{1}{4}(x - 2)$

OER from West Texas A&M, **Tutorial 15**: Equations with Rational Expressions

3) Equations with fractions (variables in the denominator)

Example: Solve $\frac{9}{3x-5} + \frac{1}{x+2} = \frac{4}{x-2}$

OER from West Texas A&M, **Tutorial 15**: Equations with Rational Expressions

4) The square root formula

If $x^2 = a$ **then** $x = \pm\sqrt{a}$,

In general, if n **a positive integer and** $x^n = a$ **(a power equation), then**

$x = \sqrt[n]{a}$, if **n is odd** and $x = \pm\sqrt[n]{a}$, if **n** is **even,** satisfy the power equation

Examples: Solve the power equations

a) $x^3 = -27$	b) $y^4 = 64$	c) $x^2 = -36$
d) $x^6 - 1 = 0$	e) $x^6 + 1 = 0$	f) $x^8 - 1 = 0$

OER from West Texas A&M, **Tutorial 16**: Formulas and Applications

5) Solving for one Variable in terms of the other

Example: $F = G\frac{mM}{r^2}$ **i)** solve for m; **ii)** solve for **r**

OER Exercise 1.2 #37 – 50: http://msenux.redwoods.edu/IntAlgText/chapter1/EquationsExercises.pdf

Quadratic Equations

Definition: An expression of the type $\boldsymbol{ax^2+bx+c=0}$ is called a **quadratic equation**.

We can solve quadratic equations by **factoring, completing the square** or by **the quadratic formula**.

1) Factoring

In **Factoring Method** we factor first. That is, to solve $\boldsymbol{ax^2+bx+c=0}$ first **factor** the **quadratic expression** $\boldsymbol{ax^2+bx+c}$ as shown

$\boldsymbol{ax^2+bx+c=\frac{1}{a}(ax+p)(ax+q)}$ where $\boldsymbol{p}$ and $\boldsymbol{q}$, if exist, are **integers** satisfying the

Sum - Product properties: $\boldsymbol{p+q=b}$ *and* $\boldsymbol{pq=ac}$

Examples: Solve by **factoring a)** $\boldsymbol{x^2-3x+4=0}$ **b)** $\boldsymbol{-3x^2+5x+8=0}$

2) Completing the square

In completing the square we write the quadratic equation in the **form** $(\boldsymbol{ex\pm h})^2=\boldsymbol{d}$, where **e, d,** and **h** are constants and $\boldsymbol{e\neq 0}$. Usually make the coefficient of $\boldsymbol{x^2}$ equal to 1; by dividing both sides of the quadratic equation with the coefficient of $\boldsymbol{x^2}$

Examples: Solve by **completing** the square

a) $\boldsymbol{x^2-6x+4=0}$ **b)** $\boldsymbol{-3x^2+15x+18=0}$

3) Quadratic Formula:

The quadratic equation $\boldsymbol{ax^2+bx+c=0}$**,** where ***a, b,* and *c*** are coefficients, can also be solved using **The Quadratic Formula**: $\boxed{\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}}$

The expression $\boldsymbol{D=b^2-4ac}$ that appears under the square root in the **Quadratic Formula** is called **the Discriminant of the** quadratic equation and is denoted by the symbol **D.**

Note: For The general quadratic Equation $\boldsymbol{ax^2+bx+c=0}, (\boldsymbol{a\neq 0})$

- If $\boldsymbol{D>0}$, then the quadratic has **two distinct real roots**
- If $\boldsymbol{D=0}$, then the equation has **exactly one root**
- If $\boldsymbol{D<0}$, then the equation **has no real solution** but has **complex conjugate roots**

Examples: Solve the following equations

a) $x^2-3x+2=0$ b)$2x^3+6x^2-18=0$ c) $3x^2-4x+3=0$

OER from West Texas A&M, **Tutorial 17:** Quadratic Equations

Other Types of Equations

- **Polynomial Equations**
- **Equations Involving Radicals**
- **Equations of Quadratic Types**

Example: **Solve the following Equations**

a) $x^4 - 3x^2 - 4 = 0$

b) $x^{2/3} - 3x^{1/3} - 4 = 0$

c) $x^5 - 25x^2 = 0$

d) $\sqrt{2 + \sqrt{x + \sqrt{2x + 1}}} = \sqrt{2 + \sqrt{2x + 2}}$

OER from West Texas A&M, **Tutorial 18**: Solving Polynomial Equations by Factoring ,
Tutorial 19: Radical Equations and Equations Involving Rational Exponents and
Tutorial 20: Equations that are Quadratic in Form

Inequalities

Important Ideas:

- **Linear Inequalities**
- **Non-linear Inequalities**
- **Sign Chart**
- **Interval Forms**

Properties of Inequalities:

1) $A \leq B \iff A \pm C \leq B \pm C$
2) **If** $C > 0$**, then** $A \leq B \iff AC \leq BC$
3) **If** $C < 0$**, then** $A \leq B \iff AC \geq BC$ **(**Multiplying by a negative number changes inequality orientation)
4) **Let** $A > 0$ **and** $B > 0$**. If** $A \leq B$ then $\frac{1}{A} \geq \frac{1}{B}$
5) **If** $A \leq B$ *and* $C \leq D$**, then** $A + C \leq B + D$

Example: Solve the following linear inequalities, **graph** the **solutions**, and give the **solutions** in **interval** and set builder forms.

a) $7x - 6 \leq 5(3x + 9) + 5$

b) $2\left(y - \frac{1}{2}\right) < 5 - 2y$

c) $-2 \leq 2 - 2x \leq 6$

d) $\frac{5}{3}x - 2 > \frac{17}{7}x + 4$

e) $-12 < -6x < 24$

OER from West Texas A&M, **Tutorial 22**: Linear Inequalities

Example: Solve the following **Nonlinear Inequalities**, graph the solutions and give the solutions in an interval and set builder forms

a) $(x-1)(2x-6) < 0$

b) $x^2 \geq 5x - 6$

c) $\frac{x+2}{x+3} < \frac{x-1}{x-2}$

d) $\frac{x-3}{x+1} \geq 0$

OER from West Texas A&M, **Tutorial 23A**: Quadratic Inequalities and **Tutorial 23B**: Rational Inequalities

Steps for Solving Nonlinear Inequalities

Sign Chart Method

1) **Make** the $\boldsymbol{right\ hand\ side} = \mathbf{0}$ (**move** all **terms** to the **left**)
2) **Factor** and **solve** for the **zeros** of all **expressions** on the **left hand side** (zeroes for **numerator** and **denominator in case of rational inequality**)
3) **Plot** the **zeros** in step **2)** on a **number line;** dividing the number line in to **intervals**
4) Select **test points from each intervals in step 2)**
5) **Plug** the **test points** for the **variables** in the **inequality** in step **1)** and decide **whether** or **not** the **inequality is satisfied**
6) **The solution set for the inequality is the union of all the interval(s) where the inequality is satisfied**

Absolute Value Equations and Inequalities

Objectives:

- **State the definition of Absolute Value**
- **Identify the different properties of absolute value equations and inequalities**
- **Identify absolute value equations and inequalities**
- **Solve absolute value equations and inequalities**

Definition: The **absolute value** of a number x **is:** $|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$

Properties of Absolute Value

1) For any number x, $|x| \geq 0$
2) For any number x and y, $|x\,y| = |x||y|$; and $|x/y| = |x|/|y|$, provided $y \neq 0$
3) For any number x and y, $|x+y| \leq |x| + |y|$
4) For any number x, $\sqrt{x^2} = |x|$

5) **Absolute Value Equations**

Let C be a non-negative number, then $|\ x\ | = C$ is equivalent to $x = C$ or $x = -C$

Example 1: Solve the following **equations**

a) $|x+3|-2=8$

b) $|3x+2|+5=1$

c) $\frac{|x+2|}{3} = 4$

d) $|x+3| = |2x+1|$

OER from West Texas A&M, **Tutorial 21**: Absolute Value Equations

6) **Absolute Value Inequalities**

Let C be a non-negative number.

a) $|\ x\ | < C$ **is equivalent to** $-C < x < C$

b) $|\ x\ | \leq C$ **is equivalent to** $-C \leq x \leq C$

c) $|\ x\ | > C$ **is equivalent to** $x < -C$ *or* $C < x$

d) $|\ x\ | \geq C$ **is equivalent to** $x \leq -C$ *or* $C \leq x$

Example 2: Solve the following equations and **graph** the solutions

a) $|\ 2x - 5\ | < 9$

b) $|\ 2x - 5\ | > 9$

c) $\left|\frac{x+1}{2} - 3x\right| \geq 4$

d) $2\left|\frac{1}{2}x + 3\right| + 3 \leq 51$

OER from West Texas A&M, **Tutorial 24**: Practice Test on Tutorials 14 - 23

Practice Problems:

OER click the link Chapter 1: Exercises with Answers (all sections combined)

Coordinates and Graphs (Page 6-14 Stitz-Zeager (S-Z) Book)

The Coordinate Plane

Objectives: **By the end of this section you should be able to**

- **Identify the coordinate plane and graph points**
- **Identify the four quadrants I, II, III, and IV**
- **Identify ordered pairs and the x- and the y- coordinates of a point**
- **Identify vertical and horizontal lines and give their equations and sketch their graphs**
- **Identify the distance formula and find distance between points in a plane**
- **Identify midpoint formula and find the midpoint of a line segment**

Quadrants: 1st, 2nd, 3rd, and 4th, Quadrants

1st – Quadrant = $\{(x,\ y): x > 0 \text{ and } y > 0\}$

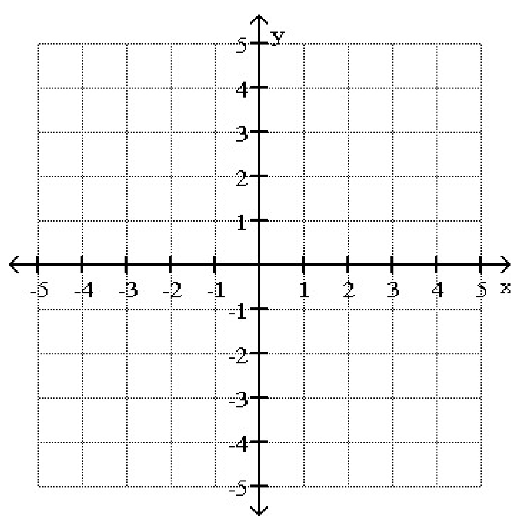
2nd – Quadrant = $\{(x,\ y): x < 0 \text{ and } y > 0\}$

3rd – Quadrant = $\{(x,\ y): x < 0 \text{ and } y < 0\}$

4th – Quadrant = $\{(x,\ y): x < 0 \text{ and } y < 0\}$

Example 1: Plot the following points on a coordinate plane.

A. (5, 2), B. (-3, 1), C. (-2, -3), D. (1, -2), E. (0, 2), F. (-1, 0)



OER from West Texas A&M, **Tutorial 25**: Slope of a Line, **Tutorial 26**: Equations of Lines
OER from West Texas A&M, **Tutorial 27**: Graphing Lines

Graphs, Table, Intercepts and Symmetries

Objectives: By the end of this section you should be able to

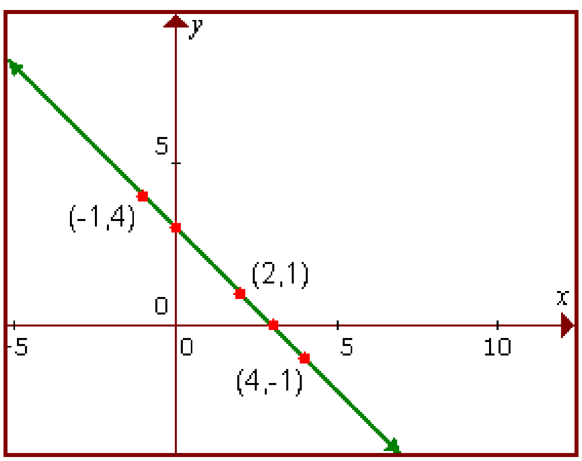
- Graph equations using table or graphing calculator
- Find x and y intercepts
- Identify three types of symmetry: symmetry with respect to the y-axis, symmetry with respect to the origin, and symmetry with respect to the x-axis.
- Test equations for symmetry
- Graph inequalities and read the domain and range from the graph

Graphs

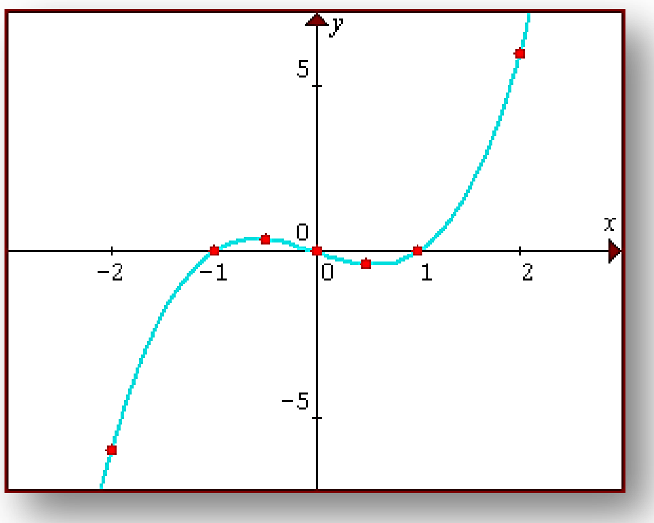
The **graph** of an equation in two variables **x** and **y** consists of the **set of points** in the **xy - plane** whose coordinates $(x,\ y)$ satisfy the given equation.

Example 1: For the graphs shown below, list some points that are on the graphs

A)



B)



Example 2: Determine if the following points are on the graph of the equation

a) $2x - y = 6$; (2, 3), (2, -2), (0, 6), (3, 0)

b) $y(x^2 + 1) = 1$; (1, 1), (1, ½), (-1, 1), (0, 1)

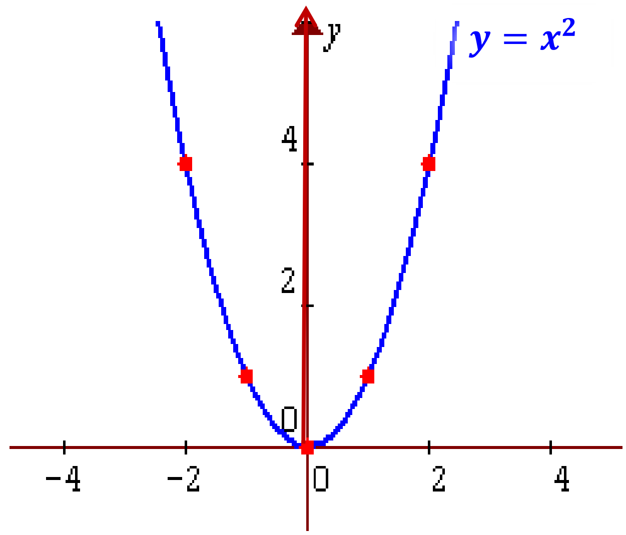
c) $x^2 + xy + y^2 = 4$; (0, -2), (1, -2), (2, -2), (1, -1)

Tables

Tables are used to help sketch graphs of equations: see **Example** 3 below

Example 3: Graph $y = x^2$ (use a table)

x	$y = x^2$	(x, y)
-3	9	(-3, 9)
-2	4	(-2, 4)
-1	1	(-1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)



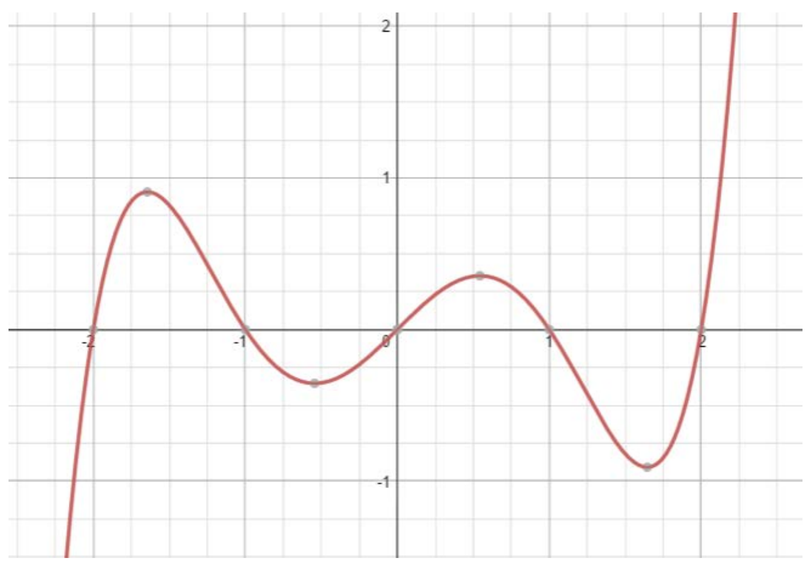
Intercepts: x - intercept and y-intercept

x - Intercepts are **points** (ordered pairs of numbers) where a **graph intersects** the **x - axis**.

y - Intercepts are **points** (ordered pairs of numbers) where a **graph intersects** the **y - axis**.

Note: At ***x* intercept y = 0** and at **y intercept *x* = 0**

Example 4: Find the intercepts form the graph



Example 5: Find the x and y intercepts:

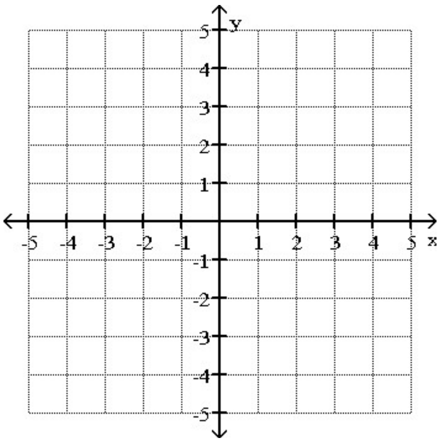
a) $y = 2x - 3$ d) $y = x^2 - 1$ g) $y - 2xy + 2x = 1$

b) $2y + 4x = 6$ e) $9x^2 + 4y^2 = 36$

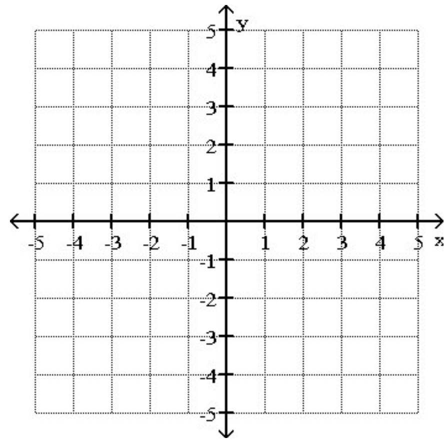
c) $y = x^2 - 5x + 6$ f) $y^2 = x^2 - 9$

Example 6: Find the **intercepts** and **sketch** graphs.

a) $3x - 2y = 6$



b) $x + y = 0$



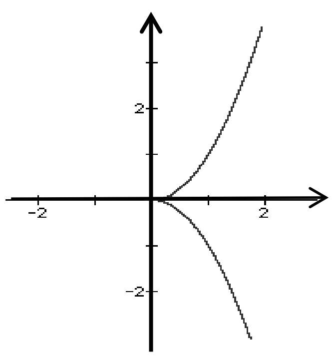
Symmetry

Symmetry with respect to **the x-axis, the y-axis, and the origin**

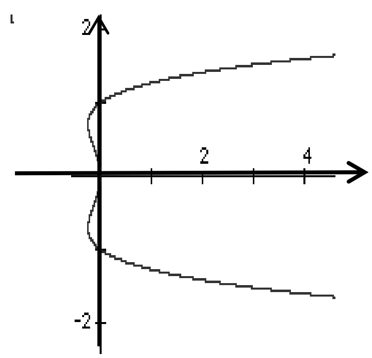
1) $\boldsymbol{x - axis}$ **Symmetry:** A graph is said to be **symmetric with respect to the x - axis if and only if** for every point (x, y) on the graph the point $(x, -y)$ is also on the graph.

Example1:

a)



b)

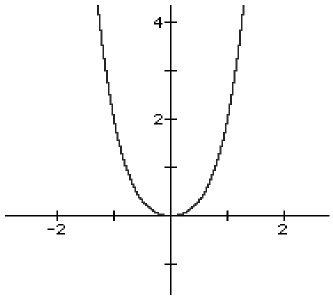


Example 2: a) $y^2 = x$

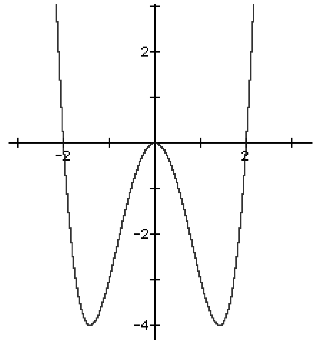
b) Unit Circle $x^2 + y^2 = 1$

2) $y - axis$ **Symmetry:** A graph is said to be **symmetric with respect to the y - axis if and only if** for every point $(x,\ y)$ on the graph the point $(-x,\ y)$ is also on the graph.

Example 3:



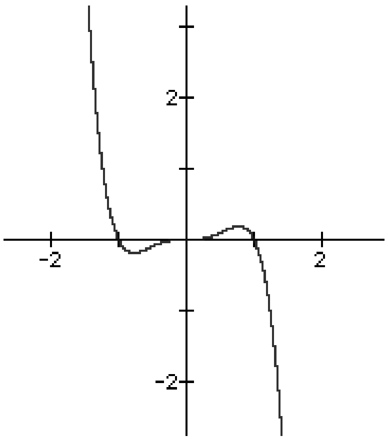
B)

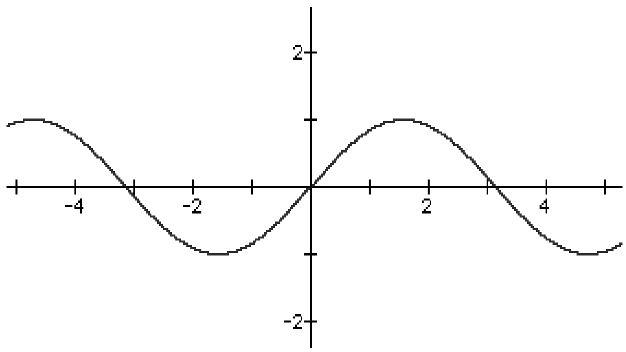


Example 4: **a)** $y = x^2$ **b)** $y = -x^2$ **c)** $x^2 + y^2 = 1$

3) **Origin (0, 0), Symmetry:** A graph is said to be **symmetric with respect to the origin if and only if** for every point $(x,\ y)$ on the graph the point $(-x, -y)$ is also on the graph.

Example 5:





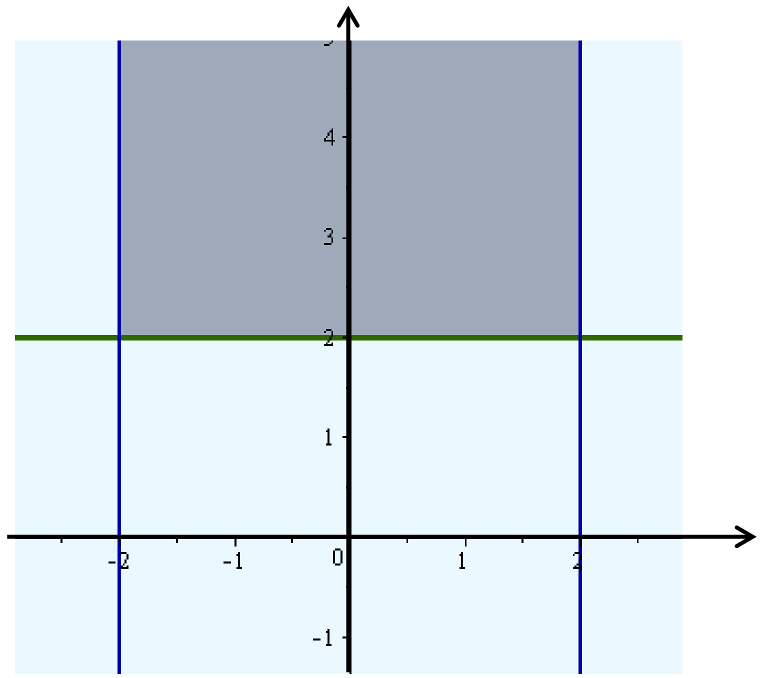
Example 6: **a)** $y = x^3$ **b)** $y = -x^3$ **c)** $x^2 + y^2 = 1$

Graphs of Inequalities in Two Variables

The graph of an inequality in two variables (usually in the variables x & y) is generally a region in the x-y coordinate axes

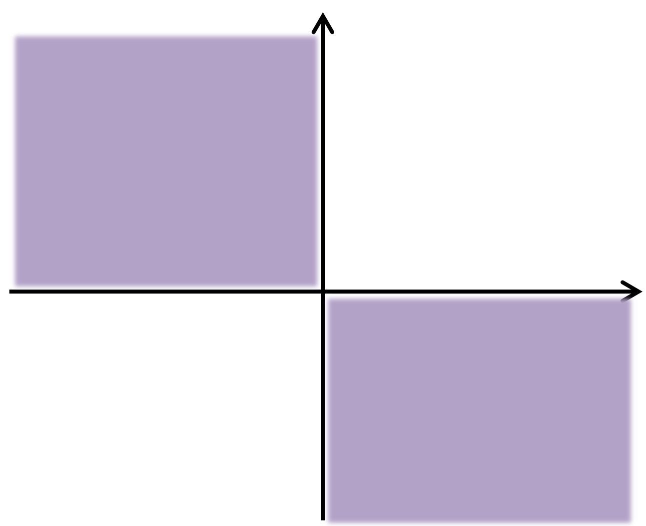
Example 1: Sketch the **region** given by the following sets and also state their **domain** and **range**:

a) $\{(x,y): -2 < x < 2 \text{ and } y \geq 2\}$



Domain $= (-2,2) = \{x: -2 < x < 2\}$,
Range$= (2, \infty) = \{y: y \geq 2\}$

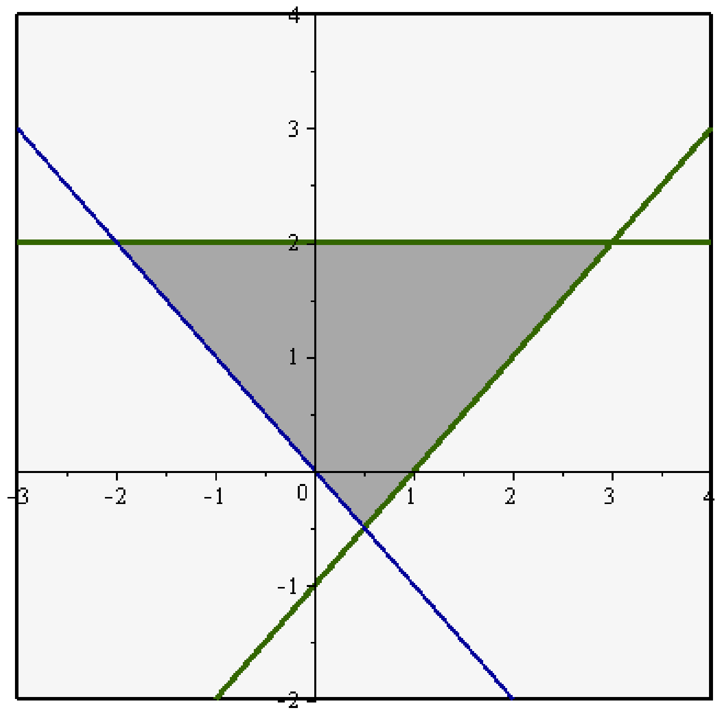
b) $\{(x,y): xy < 0\}$



$Domain = \{x: x \neq 0\}$

$Range = \{y: y \neq 0\}$

c) $\{\,(x,y):\ x+y>0,\ x-y<1,\ y\le 2\}$



$\boldsymbol{Domain}\ =\ \{x:\ -2<x<3\} = (-2,3)$

$\boldsymbol{Range}\ =\ \{y:\ -1<y\le 3\}\ =\ (-1,3]$

Example 2: Sketch the **region** defined by the following inequalities

a) $y>-1$ *and* $x>2$

b) $x>-2$

c) $y\le 4$ *and* $y\ge 1$

d) $y>x^2$ *and* $y\le 4$

e) $y\ge -2x-4, y>x+1$ *and* $y\le 2$

f) $x+y<3$

g) $2x-6y>3$ *and* $x+y<1$

Example 3: Find the domain and range of the each relation defined by each inequality in **Example** 2 above

The Distance Formula, Midpoint Formula and Circle

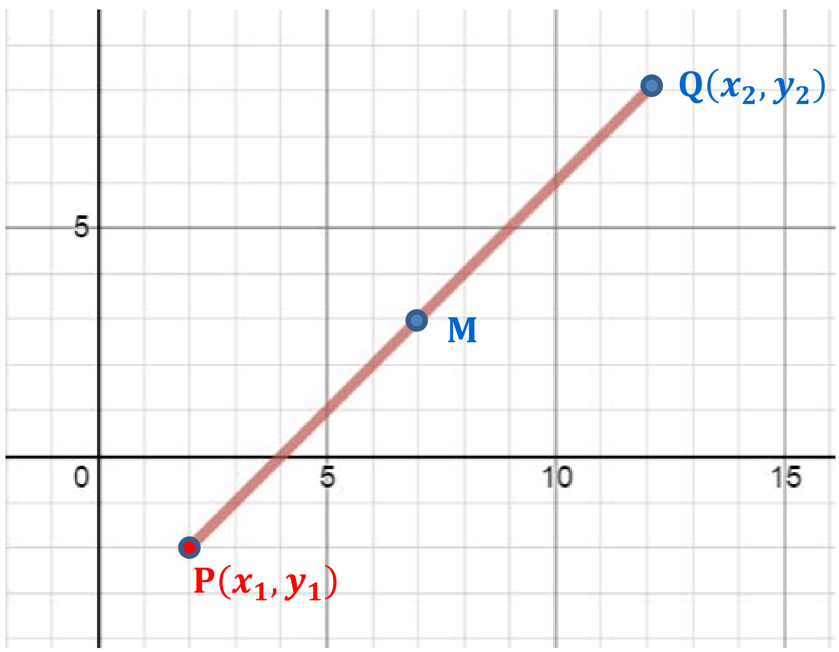
Objectives: **By the end of this section you should be able to**

- Find the distance between two points in a plane
- Find the mid-point of a line segment in the coordinate plane
- Define a circle
- Identify the standard form equation of a circle
- Graph a circle, find its center and radius
- Find the equation of a circle

The Distance and the Midpoint Formulas

Given two points $P(x_1, y_1)$ *and* $Q(x_2, y_2)$in the coordinate plane the **distance d** between **P** and **Q** and **midpoint M** of the segment **PQ** are given by:

Distance: $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; **Midpoint:** $\mathbf{M} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Example 3: Find the distance and midpoint for each pair of points

a) $(1, 2)$ & $(3, 5)$ b) $(-1, 4)$ & $(3, -2)$ c) $(4, -2)$ & $(3, 5)$

Practice Problems: S-Z book
Page 14, Exercise 1.1.4

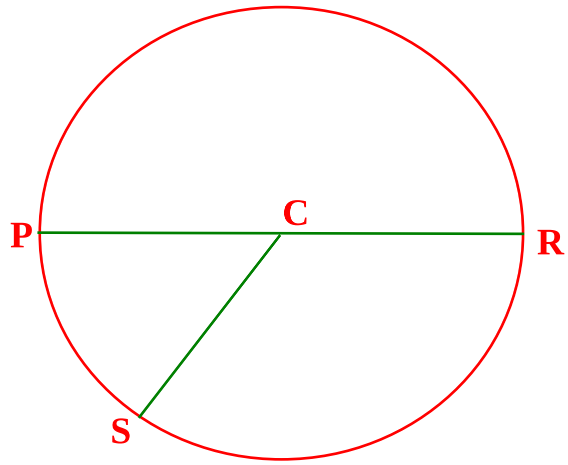
Circle

Definition: A **circle** is a set of **points** in a plane that have the **same distance** from a given point **C**.

The point **C** is called the **center** of the circle.

The chord or length **PR** through the center of a circle is called the **diameter** of the circle.

The distance **CR** **(=** **CP** **=** **CS)** from the center of a circle to the edge of the circle is called the **radius** of the circle.



Note: We can use the **Midpoint** and **Distance Formulas** to find the center and radius of a circle.

Example 1: Find the radius and center of a circle if the endpoints of the diameter are **(1, 3) and (4, 2).**

Note: **Using the distance formula we can derive the equation of a circle.**

The Equation of a Circle:

The **equation** of a **circle** with center $\boldsymbol{C} = (\boldsymbol{h}, \boldsymbol{k})$ and radius **r** is given by:

$$\boxed{(\boldsymbol{x} - \boldsymbol{h})^{\mathbf{2}} + (\boldsymbol{y} - \boldsymbol{k})^{\mathbf{2}} = \boldsymbol{r}^{\mathbf{2}}}$$

This equation is called the **Standard Form Equation** **of a circle**

Example 2: Find the center and radius of each circle

a) $(x-1)^2 + (y-3)^2 = 4$ b) $x^2 + y^2 = 1$ c) $(x+1)^2 + (y+3)^2 = 2$

Example 3: Given the center and radius, write the equation of the circle

a) Center $= (1, -3)$ and $r = 4$

b) Center = (0, -1) and $r = \sqrt{5}$

c) Center $= (-1, 0)$ and radius = 3

Example 4: Find the Equation of the circle with end points of diameter $\mathbf{P(1, -2)}$ and $\mathbf{Q(4, 5)}$

Example 5:

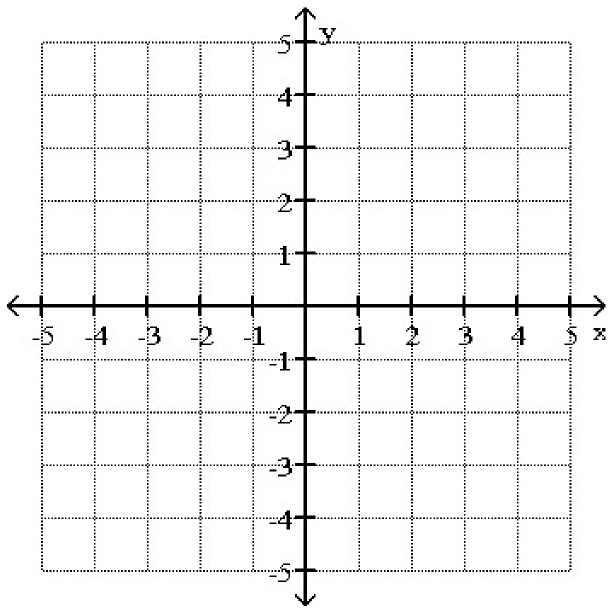
a) Show that $\boldsymbol{x^2 + 2x + y^2 - 4y - 4 = 0}$ is equation of a circle.

b) Write the following equations in the **standard form** and find the **center** and the **radius**

i. $\boldsymbol{x^2 - 3x + y^2 + 2y = \frac{9}{2}}$

ii. $\boldsymbol{3x^2 + 12x + 3y^2 - 3y - 5 = 0}$

Example 6: Graph $(\boldsymbol{x - 1})^2 + (\boldsymbol{y - 2})^2 = \mathbf{9}$



OER from West Texas A&M, **Tutorial 29**: Circles