University of North Georgia

Department of Mathematics

Instructor: Berhanu Kidane

Course: College Algebra Math 1111

Text Book: For this course we use the free e – book by Stitz and Zeager with link:

http://www.stitz-zeager.com/szca07042013.pdf

Other online resources:

e – book: http://msenux.redwoods.edu/IntAlgText/

Tutorials: http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/index.htm

For more free supportive educational resources consult the syllabus

Chapter 2
Quadratic Functions (page 151)

2.3 Quadratic Functions and Models (page 188)

Objectives: By the end of this section students should be able to
- Identify quadratic functions
- Identify the standard form of Quadratic Functions
- Find maximum and minimum points and values of Quadratic functions
- Find a vertex or turning point of a quadratic function
- Graph Quadratic Functions
- Solve applied problems that use Quadratic functions

Quadratic Functions

Definition:
A quadratic Function is a function of the form $f(x) = ax^2 + bx + c$. $a \neq 0$ where a, **b,** and **c** are called **coefficients. A quadratic function** is a **polynomial function** of degree **2.**

Example 1: a) $f(x) = x^2$

b) $f(x) = 2x^2 - 3x + 12$

Standard form of a Quadratic Function
A quadratic function $f(x) = ax^2 + bx + c$. $a \neq 0$ **using completing the form** can be expressed in the **Standard Form** $f(x) = a(x - h)^2 + k$

Example 2: **The following are in standard forms**

a) $f(x) = x^2$ b) $f(x) = -2(x - 1)^2 + 2$

The Graph of a Quadratic Function

The graph of $f(x) = ax^2 + bx + c$. $a \neq 0$ is called a Parabola which either Opens Up or

Opens **Down.** The Parabola **opens up** if $a > 0$ and **opens down** if $a < 0$

i. **Standard form:** $f(x) = a(x - h)^2 + k$
- (h, k) is called **vertex,** which is the **highest** or **lowest point**,
  - (h, k) **is the highest point if $a < 0$; and $f(h) = k$ is the maximum value**
  - **Domain $= (-\infty, \infty)$ and Range$(-\infty, k]$**
  - (h, k) **is the lowest point if $a > 0$; and $f(h) = k$ is the minimum value**
  - **Domain $= (-\infty, \infty)$ and Range$[k, \infty)$**
- The line $x = h$ is **called** the **axis of symmetry**

ii. **General form:** $f(x) = ax^2 + bx + c.$ $a \neq 0$

- **Vertex is** $\left(\frac{-b}{2a}, \ f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-b}{2a}, \ \frac{4ac-b^2}{4a}\right)$**; Vertex is also called Turning Point**

  o **If** $a > 0$**, then the minimum value is** $f\left(\frac{-b}{2a}\right)$

    o **Domain** $= (-\infty, \infty)$ **and Range**$\left[f\left(-\frac{b}{2a}\right), \ \infty\right)$

  o **If** $a < 0$**, them the maximum value is** $f\left(\frac{-b}{2a}\right)$

    o **Domain** $= (-\infty, \infty)$ **and Range**$\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$

- **The axis of symmetry is the line** $x = \frac{-b}{2a}$

- **The function** $f(x) = ax^2 + bx + c.$ $a \neq 0$ has the standard form

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

Example 3: For the graphs of the parabolas below find the vertex, the maximum or the minimum point, the maximum or the minimum value, and the axis of symmetry

a)
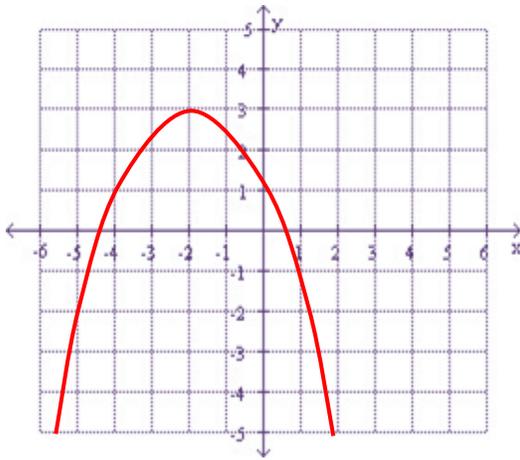
b)
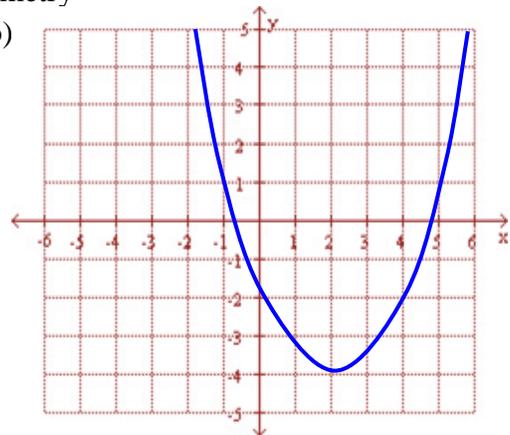
Examples: **Example 2.3.1 Page 188 Reading**

Example 2.3.2 Page 190 Reading

Example 4: Write the following in the standard form

a) $f(x) = 2x^2 - 12x + 23$ b) $f(x) = 3x^2 - 6x + 1$

OER 1: West Texas A&M University Tutorial 34: Graphs of Quadratic Functions

OER 2: Notes and Exercises
a) Chapter 5: Quadratic Functions
    a. Section 1: The Parabola
        i. Exercises with Answers
    b. Section 2: Vertex Form
        i. Exercises with Answers
    c. Section 5: Motion   (#29 – 40 Odd numbered problems)
        i. Exercises with Answers
    d. Section 6: Optimization (#1 – 38 odd numbers)
        i. Exercises with Answers

3 ways to find **a vertex or turning point** of a quadratic function:

1) If in the standard form, $f(x) = a(x - h)^2 + k$, then $\textbf{vertex} = (h, k)$

2) If **not** in the standard form, then use the formula $\left(\frac{-b}{2a}, \ f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-b}{2a}, \ \frac{4ac - b^2}{4a}\right)$.

3) **Completing** the **square** if **not in standard form**.

Example 5: Find the vertex, maximum and minimum values and the range of each.

a) $f(x) = x^2 + 10x + 23$ b) $f(x) = (x - 2)^2 + 3$

Example 6: Find the vertex, axis of symmetry, the minimum or maximum point, intervals where increasing and decreasing, and graph each.

a) $f(x) = (x + 3)^2 - 1$ b) $f(x) = -2x^2 + 8x - 6$

Example 2.3.3: Page 196 Reading

Example 2.3.4: Page 197 Reading

Example 7: Suppose $C(x) = x^2 - 10x + 27$ represents the costs, in hundreds, to produce x thousand pens. How many pens should be produced to minimize the cost? What is this minimum cost?

Example 8: A stone is thrown directly upward from a height of 30 ft with an initial velocity of 60 ft/sec. The height of the stone t seconds after it has been thrown is given by the function $S(t) = -16t^2 + 60t + 30$. Determine the time at which the stone reaches its maximum height and find the maximum height.

Example 9: Skippy wishes to plant a vegetable garden along one side of his house. In his garage, he found 32 linear feet of fencing. Since one side of the garden will border the house, Skippy doesn't need fencing along that side. What are the dimensions of the garden which will maximize the area of the garden? What is the maximum area of the garden?

Example 10: What is the largest rectangular area one can enclose with 14 inches of string?

Example 11: The height of an object dropped from the roof of an eight story building is modeled by $h(t) = -16t^2 + 64, \ 0 \leq t \leq 2$. Here, h is the height of the object off the ground, in feet, t seconds after the object is dropped. How long it takes before the object hits the ground?

Example 11: Carl's friend Jason participates in the Highland Games. In one event, the hammer throw, the height h in feet of the hammer above the ground t seconds after Jason lets it go is modeled by $h(t) = -16t2 + 22.08t + 6$. What is the hammer's maximum height? What is the hammer's total time in the air? Round your answers to two decimal places..

Practice Problems from the Text
Page 200, Exercise 2.3.1: 1 – 34 Odd numbers

Chapter 3
Polynomial Functions (page 235)

3.1 Polynomial Functions and Their Graphs

Objectives: By the end of this section students should be able to:
- **Identify polynomial Functions**
- **Graph basic polynomial functions**
- **Identify end behavior and leading term**
- **Use zeros in graphing polynomials**
- **Find local maximum and local minimum**

Definition of a Polynomial Function
A **polynomial function** of **degree** n **is a function of the form**

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$ where n **is a non-negative integer** and $a_n \neq 0$

- The numbers $a_n, a_{n-1}, \ldots a_1, a_0$ are called the **coefficients** of the polynomial
- The number a_0 is called the **constant coefficient** or **constant term**
- The number a_n, the coefficient of the highest power, is the **leading coefficient**
- The term $a_n x^n$ is the **leading term**
- $p(x) = a_0$ is a polynomial of **degree 0**
- If $p(x) = 0$, we say **P** has **no degree**

Example 3.1.1 Page 235 reading: Which of the following functions are polynomials/

Example 1: Which of the following is a polynomial? If an expression is a polynomial, **name its degree**, and tell the variable that the polynomial is in.
- a) $f(x) = x^3 - 2x^2 - 3x - 4$
- b) $p(y) = 3y^2 + 2y + 1$
- c) $h(x) = x3 + 2\sqrt{x} + 1$
- d) $f(x) = \sqrt[3]{x^2}$
- e) $f(z) = z^{-1} + 2$

Example 2: $p(x) = -2 x^5 + x^4 + 3x^3 - 4x^2 - 12x + 6$
- a) **Degree**
- b) **Leading term**
- c) **Leading coefficient**
- d) **Constant term**

Example 3.1.2 Page 237 Reading

Example 3: Give examples of polynomial function of
- a) **degree 0**
- b) **degree 1**
- c) **degree 2**
- d) **degree 33**

Example 3.1.3 Page 338: An open box will be made by cutting out congruent squares from each corner of a 3ft by 8ft piece of cardboard and then folding up the sides. Let x denote the length of the side of the square which is removed from each corner.

    a) Find the volume V of the box as a function of x. Include appropriate applied domain

    b) Using graphing calculator graph $y = V(x)$ on the domain from 1) and approximate the maximum volume to two decimal places. What is the maximum volume?

Graphs of Basic Polynomials

The simplest polynomial functions are the monomials $p(x) = ax^n, a \neq 0$

Example 4: Using graphing utilities sketch the graph of $y = ax^n$, for $n = 1, 2, 3, 4, 5$ and so on and deduce some general properties of the graphs of $y = ax^n$

**Recall:** The coefficient a in $y = ax^n$, for $a > 0$ either **stretches** or **shrinks** the graph vertically. If $a < 0$ the graph of $y = ax^n$ is a **reflection** of $y = |a|x^n$ across the x-axis. Therefore, we consider graphs of $y = ax^n$ for the cases where $a = \pm 1$. **Two cases** $i) \, n - odd \, and \, ii) \, n - even$

i) $n - odd$; $n = 1, 3, 5, 7$ ...

    a) $a > 0$ End behaviour**: DOWN, UP**   **b)** $a < 0$ End behaviour**: UP, DOWN**

$y = x^n$
$n = 1, 3, 5, 7$

$y = -x^n$,
$n = 1, 3, 5, 7$
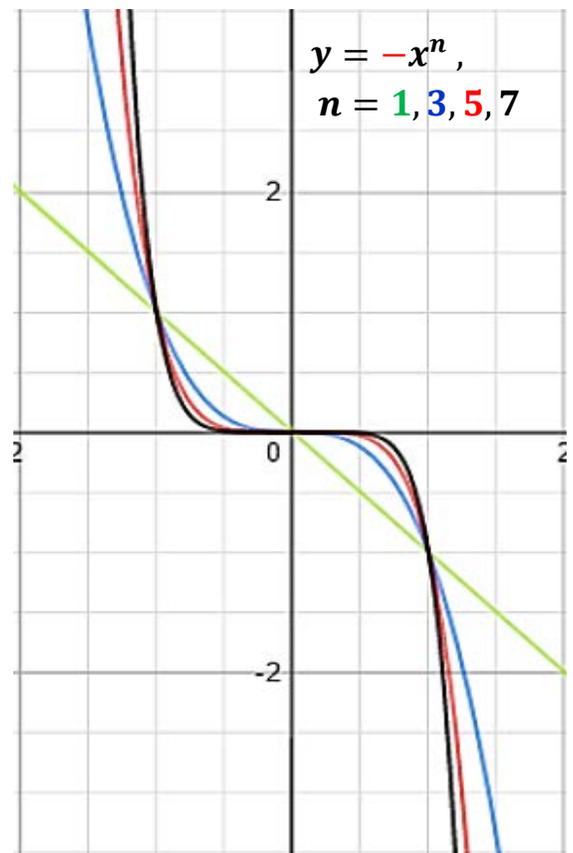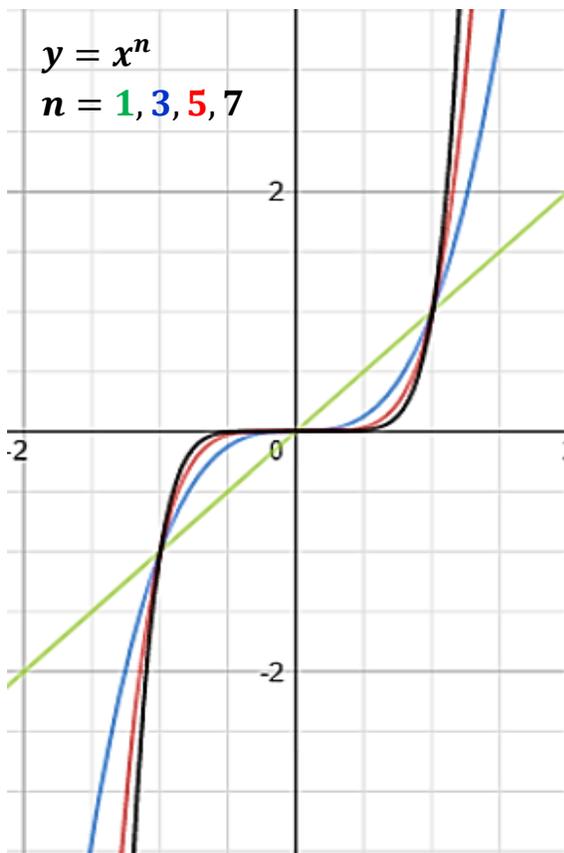
ii) $n - even$; $n = 2, 4, 6, \ 8 \dots$

    a) $a > 0$ End behaviour: **UP, UP**   **b)** $a < 0$ End behaviour: **DOWN, DOWN**
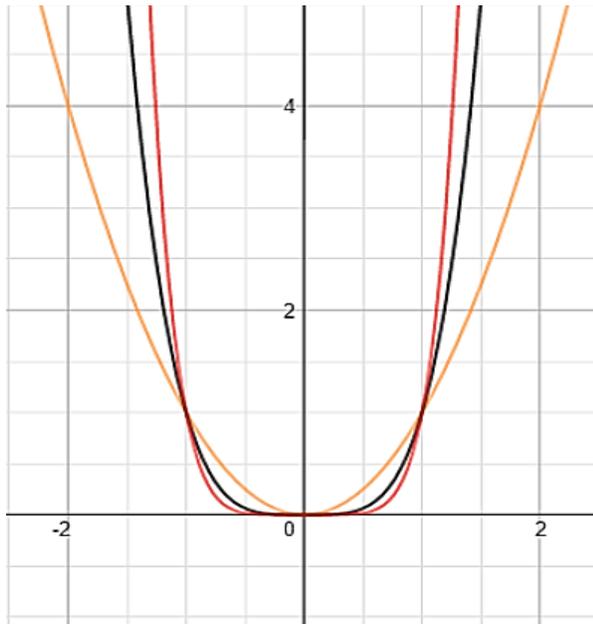
      $y = x^n$, $n = 2, 4, 6$ $y = -x^n$, $n = 2, 4, 6$
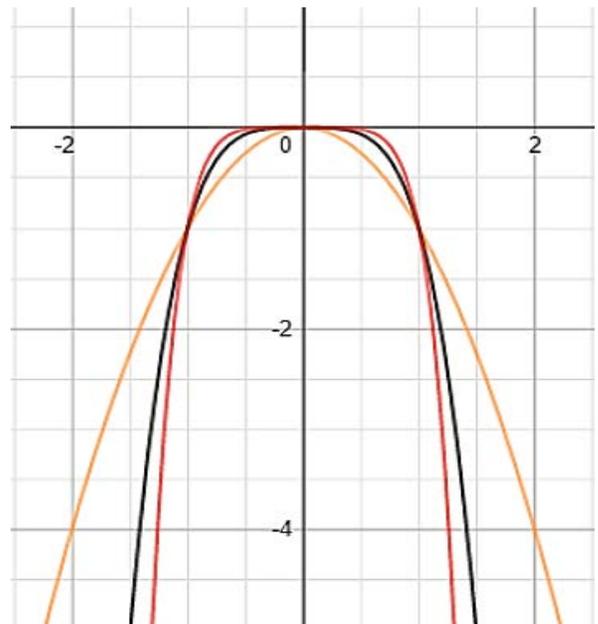
Let **P** be a polynomial function of **degree n.**

1.   The domain of **P** (any polynomial function) is the **set of all real numbers**

2.   **P** is continuous for all real numbers, so there are no **breaks**, **holes**, or **jumps** in the **graph**.

3.   The graph of **P** is a **smooth curve** with rounded corners and no sharp corners or cusps.

4.   The graph of **P** has **at most *n x-intercepts* or *n-zeroes***.

5.   The graph of **P** has **at most n – 1 turning points**.

6.   The graph of **P** has f**our** types of **end behaviour**s

Example 5: Sketch the graph of the following functions using graphing utilities and find their end behavior, number of zeros and turning points

    a) $f(x) = x^3 - 2x^2 - 3x + 2$

    b) $f(x) = -x^3 - 2x^2 + 3x + 2$

    c) $f(x) = -x^4 + 4x^2 - 1$

    d) $f(x) = x^4 - 4x^2 + 1$

    e) $P(x) = x^2(x + 2)(x + 1)(x - 1)(x - 2)$

**Example 6:** Each of the following graphs **cannot** be a graph of a polynomial function; Why?.

A)

B)

C)

D)
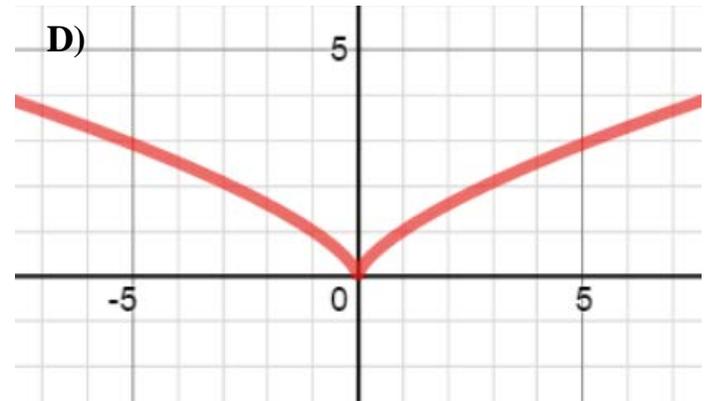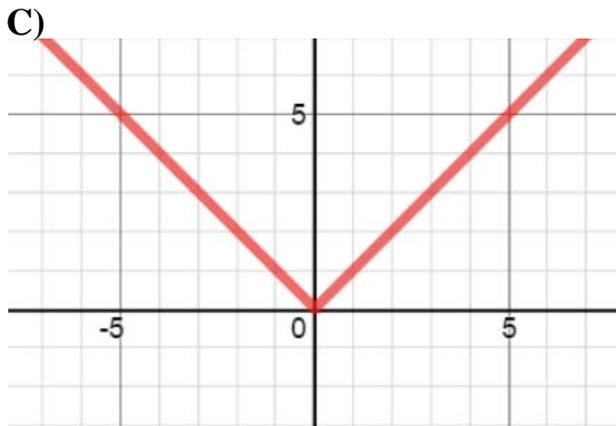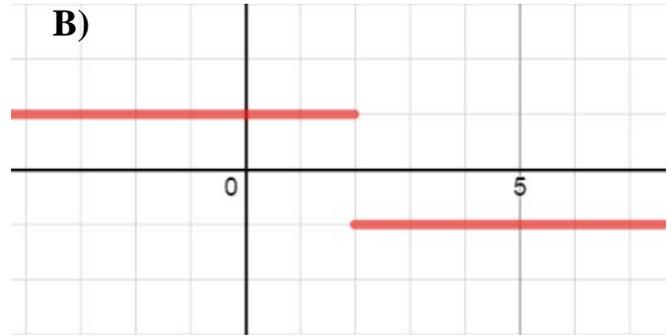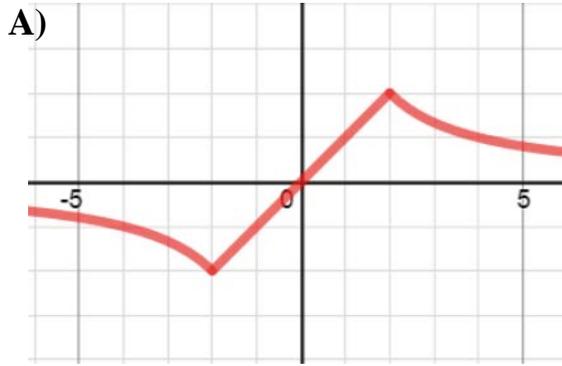
Let $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a **polynomial function of degree n**

Table 1 shows **End Behaviors** for the graphs of **polynomial functions** of **Degree n** and **Leading Coefficient a_n**

Leading Coefficient / Degree	End Behavior
$a_n > 0$ n **even**	both ends up (up, up)
$a_n < 0$ n **even**	both ends down (down, down)
$a_n > 0$ n **odd**	left down, right up (down, up)
$a_n < 0$ n **odd**	left up, right down (up, down)

Table 1

Example 7: Classifying Polynomials by Their Graphs

i.   **Even Degree**
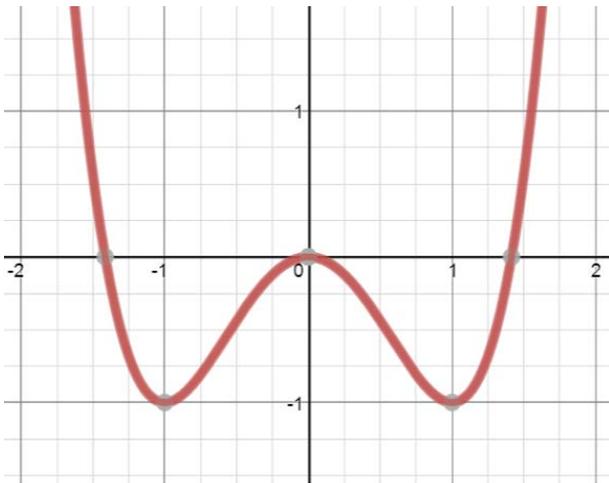

$$y = x^4 - 2x^2$$
$a_n > 0$, opens up or **up, up**
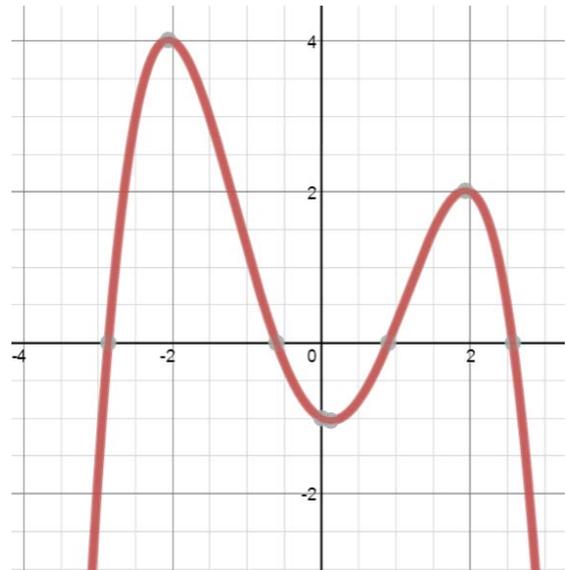
$$y = -\frac{1}{4}x^4 + 2x^2 - \frac{1}{2}x - 1$$
$a_n < 0$, opens down or **down, down**

ii.   **Odd Degree**
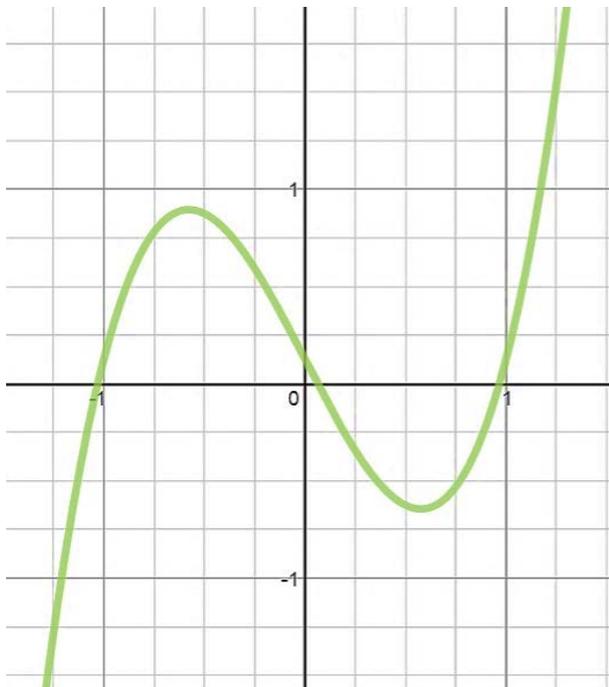
$$y = 2x^3 - 2x + \frac{1}{8}$$
$a_n > 0;$ **Left Down, Right UP**
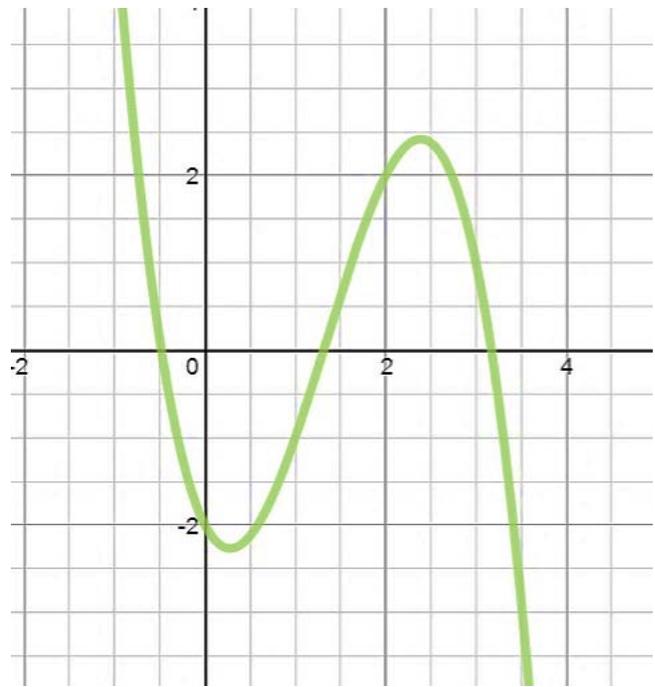
$$y = -x^3 + 4x^2 - 2x - 2$$
$a_n < 0;$ **Left UP, Right Down**

Example 8: Determine the **leading term**, the **leading coefficient**, the **degree** of the **polynomial**, and the **end behavior** of the graph.

a) $f(x) = 2x^3 + 3x^2 - 5x + 4$

Ans. 1) leading term = $\mathbf{2x^3}$

    2) Leading coeff. = **2**

    3) Degree = **3**

    4) End Behavior **down, up**

b) $f(x) = -x^4 + 2x^3 + 3$

Ans. 1) Leading term $= -\mathbf{x^4}$

    2) Leading coeff. = **–1**

    3) degree = **4**

    4) End Behavior: **down, down**

c) $f(x) = -x^5 + 3x^3 + 7$

d) $f(x) = 2x^2 + 3x + 2$

Example: Name the **degree**, the **leading coefficient**, and the **constant term** of
$$h(x) = (5x + 1)(3x - 1)(2x + 5)^3$$

Solution:
If we were to multiply out, then the **degree** of the **product** would be the **sum** of the **degrees** of **each factor**, thus the degree of $h(x) = 1 + 1 + 3 = 5$.
Note: $h(x) = (5x + 1)(3x - 1)(2x + 5)^3 = (5x + 1)(3x - 1)(2x + 5)(2x + 5)(2x + 5)$.
The **leading coefficient** would be the **product** of all the **leading coefficients**: $5 \cdot 3 \cdot 2^3 = 15 \cdot 8 = 120$.
And the **constant term** would be the **product** of all the **constant terms**: $1 \cdot (-1) \cdot 5^3 = -1 \cdot 125 = -125$.

Example 9: Find the degree, the leading coefficient, and the constant term.

a) $f(x) = 6x^3 + 7x^2 - 3x + 1$

b) $f(x) = (x - 1)(x^2 + x - 6)$

b) $g(x) = (x + 2)^2(x - 3)^3(2x + 1)^4$

c) $h(x) = x(x - 2)^5(x + 3)^2$

d) $f(x) = 5x^3 - 4x^2 + 7x - 8$

Example 10: Referring to the graphs below:

   i.   Identify as **even** or **odd** degree **polynomials**

   ii.   Determine **possible degrees** and **signs of leading coefficients**

   iii. Find possible **zeros** of the **polynomials.**

a)

b)
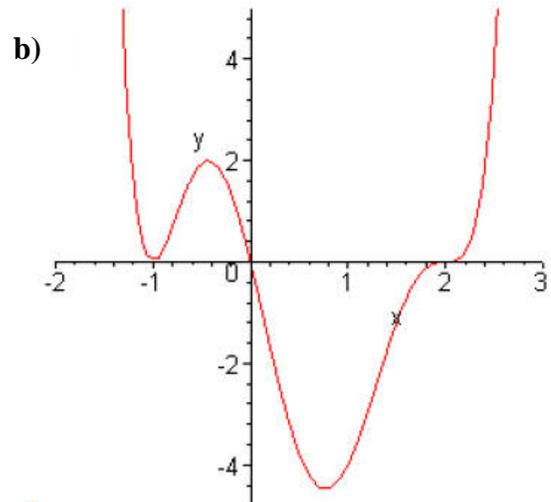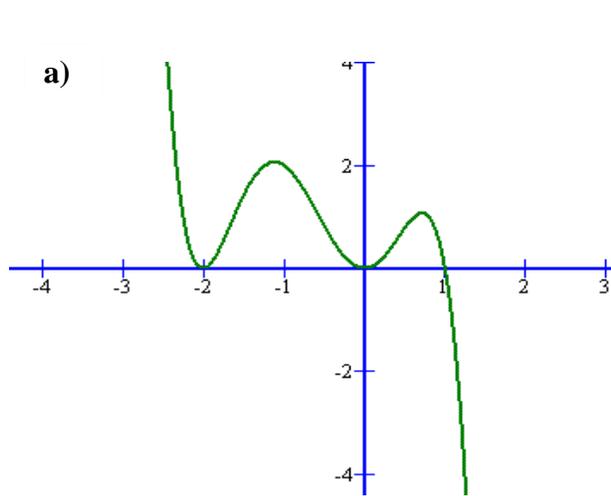
The zeroes of a polynomial $y = p(x)$ and Multiplicity

Zeroes of a polynomial

Recall: If $p(r) = 0$ for a number r, then r is called the **zero** of p, to find the zeros of p:

- Set $p(x) = 0$ and solve for x.

- Factor, if it is possible to factor, the polynomial p

Example 11: Find the zeros of $f(x) = x^3 + 2x^2 - 5x - 6$.

      Factor $f(x)$ and set it **equal to 0** and **solve** for **x.**

$$x^3 + 2x^2 - 5x - 6 = (x + 3)(x + 1)(x - 2)$$

$$x = -3, -1, \text{ and } 2, \text{ are the zeros of the function.}$$

Note that: $f(-3) = 0, \ f(-1) = 0, \text{ and } f(2) = 0$

Multiplicity

Definition (Multiplicity)

The **multiplicity** of a zero is the number of times that zero occurs. For the polynomial
function $f(x) = (x - c)^k$, c is a **zero** of the function with **multiplicity** k.

- If **k** is **odd**, then the graph **crosses** the x-axis at **(c, 0)**

- If **k** is **even**, then the graph **is tangent to** the x-axis at **(c, 0)** (touches the x-axis but does
  not cross it)

Theorem: Suppose **P** is a polynomial function and $x = c$ is a zero of multiplicity m. Then:

- If **m** is even, then the graph of **P is tangent to** the x-axis at **(c, 0)** (touches and re-bounce from the x-axis at **(c, 0)**)

- If m is **odd**, then the graph of **P crosses** the *x*-axis at **(c, 0)**

Example 14: **For each of the following find the zeroes, state the multiplicity, and sketch the graph**

    a) $f(x) = 5x(x - 2)^3(x + 1)$

Solution: $x = 2$ is a zero with *multiplicity 3*; *(graph crosses the at $x = 2$)*
        $x = -1$ is a zero with **multiplicity 1;** *(graph crosses the at $x = -1$)*
        $x = 0$ is a zero with multiplicity 1; *(graph crosses the at $x = 0$)*

    b) $f(x) = -x^2(x - 1)^3(x + 2)^4$.

Solution: $x = -2$ is a zero with *multiplicity 4*; *(graph re-bounces at $x = -2$)*
        $x = 1$ is a zero with **multiplicity 3;** *(graph crosses the at $x = 1$)*
 $x = 0$ is a zero with multiplicity 2; *(graph re-bounces at $x = 0$)*

**Example 3.1.6 page 245:** Reading

Important ideas for sketching graphs of polynomials:

- **Zeros and their multiplicity**

- **Degree and leading coefficient**

- **End Behavior:** The **leading term** and the **degree** tells us about the **end behavior**

- **Intercepts: x and y - intercepts**

- **Symmetries:** If any

- **Test Points:** Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the x-axis on the intervals determined by the zeros. Include the y-intercept on the table.

- **Graph:** Plot the intercept and other points you found on the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

Example 12: Choose the correct graph of $h(x) = -x(x-4)(x+1)(x-5)$
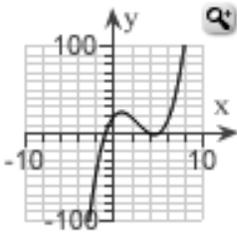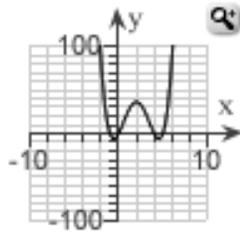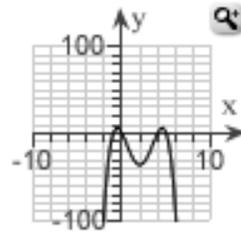
○ A.　　　　　　○ B.　　　　　　○ C.　　　　　　○ D.
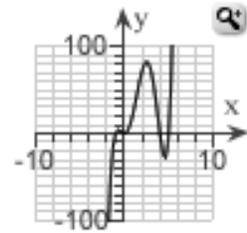
Example 13: Choose the correct graph of $f(x) = x^5 + 3x^2$
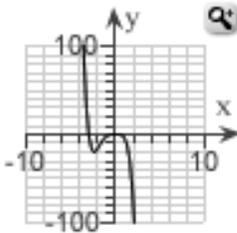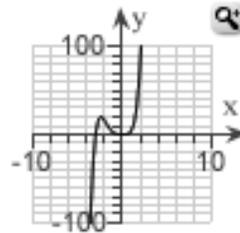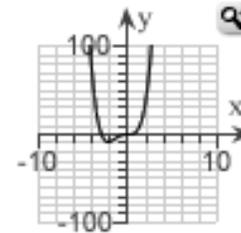
○ A.　　　　　　○ B.　　　　　　○ C.　　　　　　○ D.
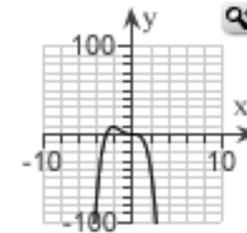
Example 15: Sketch the graph of the following polynomials

a) $f(x) = 6x^3 + 7x^2 - 3x + 1$

b) $f(x) = (x-1)(x^2 + x - 6)$

c) $g(x) = x(x+2)(x-3)^2(2x+1)$

d) $h(x) = -(x-1)(x-3)(x+2)(x+1)$

OER 1: West Texas A&M University Tutorial 35: Graphs of Polynomial Functions

OER 2: Notes and Exercises College of the Redwoods:
Chapter 6: Polynomial Functions
- Section 1: Polynomial Functions
  - Exercises with Answers
  - Exercises with Solutions
- Section 2: Zeros of Polynomials
  - Exercises with Answers
  - Exercises with Solutions
- Section 3: Extrema and Models
  - Exercises with Answers
  - Exercises with Solutions

Practice Problems from the Text
Page 246, Exercises 3.1.1: #1 – 32 (odd numbers)

Polynomial Divisions (Page 257)

Long Division and Synthetic Division

Long Division

The Division Algorithm

If $P(x)$ and $D(x)$ are polynomials, with $D(x) \neq 0$, then there are **unique polynomials**

$Q(x)$ and $R(x)$, where $R(x)$ is **either 0** or of **degree less** than the **degree** of $D(x)$, such that

$$P(x) = D(x) \cdot Q(x) + R(x)$$

The polynomials $P(x)$ and $D(x)$ are called the **Dividend** and **divisor respectively**

$Q(x)$ **is** called the **quotient**

$R(x)$ is called the **remainder**

For Example: If we divide $6x^2 - 26x + 12$ *by* $x - 4$ we get

$$6x^2 - 26x + 12 = (x - 4)(6x - 2) + 4$$

In the **Division Algorithm** Format:

$P(x) = 6x^2 - 26x + 12$ is the **Dividend;**

$D(x) = x - 4$ is the **Divisor**;

$Q(x) = 6x - 2$ is the **Quotient** and

$R(x) = 4$ is the **Remainder**

Example 1: Divide $(x^4 - 2x^2 + x - 2) \div (x^2 + x - 4)$

Solution: By **Division Algorithm**:

$$x^4 - 2x^2 + x - 2 = (x^2 + x - 4) \cdot Q(x) + R(x)$$

Where $Q(x)$ and $R(x)$ **are polynomials** to be determined using **Polynomial long Division**

In dividing polynomials using **Long Division**:
First we **must insert zero** placeholders for missing terms and rewrite the division as:

$$(x^4 + 0x^3 - 2x^2 + x - 2) \div (x^2 + x - 4)$$

Next, set up the polynomial division as a standard division problem and repeat the steps Divide, Multiply, Subtract, Carry Down over and over until the divisor no longer may be divided into the result at the bottom.

Step 1: We eliminate x^4 from the **dividend**, to do so we need to multiply the **divisor** by x^2 and subtract the product from the dividend and bring down x to get a **new dividend,** $-x^3 + 2x^2 + x$

Step 2: Next we eliminate $-x^3$ from **the new dividend,** to do so multiply the **divisor** by $-x$ and subtract the product from $-x^3 + 2x^2 + x$ and bring down -2 , which gives a **second new dividend** $3x^2 - 3x - 2.$ Repea**t this process** for the **new dividend,** until we get a **dividend** of **degree smaller** than the **divisor,** $x^2 + x - 4$

$$
\begin{array}{r}
x^2 - x + 3 \\
x^2 + x - 4 \overline{)\, x^4 + 0x^3 - 2x^2 + x - 2} \\
\underline{x^4 + \ x^3 - 4x^2} \\
0 - \ x^3 + 2x^2 + x \\
\underline{-x^3 - \ x^2 + 4x} \\
0 + 3x^2 - 3x - 2 \\
\underline{3x^2 + 3x - 12} \\
-6x + 10
\end{array}
$$

Since, $-6x + 10$ is of smaller degree than $x^2 + x - 4,$ we stop the process here.

The polynomial $x^2 - x + 3$ is **Quotient,** and $-6x + 10$ is **reminder; and so,**

$$x^4 - 2x^2 + x - 2 = (x^2 + x - 4) \cdot (x^2 - x + 3) + (-6x + 10)$$

Example 2: Using Long Division, find the quotient and the remainder of each of the following.

a) $f(x) = \dfrac{3x^3 + 2x - 4}{x^2 - 4}$

b) $f(x) = \dfrac{5x^4 + 3x^2 + 2x - 8}{2x^2 + 2x - 8}$

c) $f(x) = \dfrac{x^4 + 2x^3 - 6x^2 - 2}{x - 1}$

OER: West Texas A&M University Tutorial 36: Long Division

Homework Practice Problems from the Text
Exercise 3.2.1 Page 265 #1 – 6

Synthetic Division - The Shortcut for Dividing by $(x - c)$

When dividing a polynomial $f(x)$ by a linear factor $(x - c)$, we can use a shorthand notation saving steps and space.

Procedure for Synthetic Division; we proceed with example

Example 3: Divided $f(x) = 3x^3 + 2x - 1$ by $(x - 4)$.

1. Insert **zero** place holder for the missing term: $f(x) = 3x^3 + 0x^2 + 2x - 1$

2. Write the value of "**c**" and the coefficients of $f(x)$ in a row. In **Example 3 c = 4**, and the coefficients are **3, 0, 2,** and **-1.**

    ```
    4 | 3  0  2  -1
      |_____
    ```

3. Carry down the first coefficient. In this case carry down the **3**.

    ```
    4 | 3  0  2  -1
      |
      3
    ```

4. Multiply this carried down coefficient by the value of c.
    In this case, multiply **3 • 4 = 12**. Place this result in the next column.

    ```
    4 | 3  0  2  -1
      |   12
      3
    ```

5. Add the column entries and place result at bottom. In this case you add **0 + 12** to get **12.**
    Multiply this addition result by "c" and place in next column. In this case you multiply **12 • 4 = 48**

    ```
    4 | 3  0  2  -1
      |   12 48
      3  12
    ```

6. Repeat Step 4 for all columns. In this example, you get

    ```
    4 | 3  0  2  -1
      |   12 48 200
      3  12 50 199
    ```

7. The bottom row of numbers reveals the answer along with the remainder. In this case, the numbers **3 12 50 199** indicate an answer of
    $3x^2 + 12x + 50$, **with remainder 199 or** $3x^2 + 12x + 50 + 199/(x - 4)$

Note: The answer will always have degree one less than the dividend.

Example 4: **Using synthetic division, find the quotient and remainder**

a) $f(x) = \dfrac{x^4 + 2x^3 - 6x^2 - 2}{x - 1}$

b) $x^5 + 32 \div x + 2$

Example 3.2.1 Page 261: Reading

Homework Practice Problems from the Text
Exercise 3.2.1 Page 265 #7 – 20

The Factor Theorem:

For a polynomial $f(x)$, if $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

Note: $x - c$ is a factor of $f(x)$ means, the remainder when $f(x)$ divided by $x - c$ is **0**

Example 5: Let $f(x) = x^3 - 2x^2$.

$f(2) = 0$, so by the **Factor Theorem,** $x - 2$ is a factor of $f(x)$

Example 6: Let $f(x) = x^3 + 2x^2 - 5x - 6$

a) Use **long division** to determine whether $x + 3$ and $x - 3$ are factors of $f(x)$.

b) Use **The Factor Theorem** to determine whether $x + 3$ and $x - 3$ are factors of $f(x)$.

c) Use **synthetic division** to determine whether $x + 3$ and $x - 3$ are factors of $f(x)$.

The Remainder Theorem:

If $f(x) = (x - c)Q(x) + R$, then $f(c) = R$. That is, the **remainder** when dividing the polynomial $f(x)$ by $x - c$ is the same as the **value** of the **function** evaluated at $x = c$.

Example 7: Using the **Remainder Theorem**, find the remainder when $f(x) = x^3 + 2x^2 - 5x - 6$ is divided by:

a) $x + 2$

b) $x - 1$

Example 3.2.2 Page 362: Reading

Example 8: Decide whether the numbers $-3, \ 2,$ are zeros of the polynomial

$\quad f(x) = 3x^3 + 5x^2 - 6x + 18$; use both **Synthetic Division** and the **Remainder Theorem**

Example 9: Factor the polynomial $f(x) = x^3 + 5x^2 - 2x - 24$ and solve the equation $f(x) = 0.$

Solution:

1) First, list all integral factors of -24: which are $\pm 1, \ \pm 2, \ \pm 3, \ \pm 4, \ \pm 6, \ \pm 8, \ \pm 12, \ \pm 24$

2) Next, check if any of these factor is a zero of $f(x)$

    Check for: $f(\pm 1) =?, \ f(\pm 2) =?, \ etc.$

3) Finally using the result of 2) and division of polynomials factor $f(x)$

Example 10: Factor the polynomial $f(x) = x^4 + 2x^3 - 25x^2 - 50x$ completely

Example 11: Factor the polynomial $f(x) = x^4 - 8x^4 - 33$ completely

Example 12: Factor the polynomial $f(x) = x^4 - 5x^3 + 20x - 16$ completely

Example 12: Solve $x^3 + 4x^2 + 25x - 100 = 0$

OER: West Texas A&M University
    Tutorial 37: Synthetic Division and the Remainder and Factor Theorems

OER: West Texas A&M University on zeros of polynomial functions
    Tutorial 38: Zeros of Polynomial Functions, Part I
    Tutorial 39: Zeros of Polynomial Functions, Part II

Homework Practice Problems from the Text
    Exercise 3.2.1 Page 265 #21 – 46 (odd numbers)