Math 1111 – Factoring Polynomials

Objectives:

1. Factor the greatest common factor of a polynomial

2. Factor by grouping

3. Factor a trinomial where the leading coefficient is 1

4. Factor a trinomial in the form ax2 + bx + c

a. Using the trial and error method

b. Using the “ac” method

5. Factor a perfect square trinomial

6. Factor a difference of squares

7. Factor the sum and difference of cubes

The greatest common factor (GCF) of two numbers is the largest number that divides evenly into two or more numbers. For example, 4 is the GCF of 16 and 20 because it is the largest number the divides evenly into both numbers. The GCF of polynomials works the same way. For example, 4x is the GCF of 16x and 20x2 because it is the largest polynomial that divides evenly into both terms. When factoring a polynomial expression, always look for the GCF of the coefficients and then the variables before you begin to factor.

**Objective 1:** Factor out the greatest common factor (GCF) of a polynomial expression.

1. Identify the GCF of the coefficients.

2. Identify the GCF of the variables.

3. Combine to find the GCF of the expression.

4. Determine what the GCF needs to be multiplied by to obtain each term in the expression.

5. Write the factored expression as the product of the GCF and the sum of the terms we need to multiply by.

Example #1: Factor the greatest common factor (GCF).

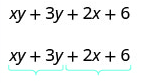
A. 6x3y3 + 45x2y2 + 21xy

B. 15x3 – 25x2

C. x (b2 – a) + 6(b2 – a)

When a polynomial has four terms, you can use the grouping method. You will want to separate the polynomial into two parts with two terms in each part. Then look for the GCF in each part. If the polynomial can be factored, you will find that a common factor emerges from both parts.

Factor the polynomial xy + 3y + 2x + 6 by grouping.



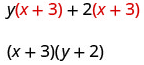
Separate the first two terms from the second two terms.



Factor the GCF from the first two terms.



Factor the GCF from the second two terms.



Notice that each term has a common factor of (x + 3).

Factor out the GCF, which is (x + 3)

**Objective 2:** Factor by grouping.

1. Group terms with common factors.

2. Factor out the common factor in each group.

3. Factor the common factor from the expression.

4. Check by multiplying the factors or using the FOIL method.

Example #2: Factor the polynomials by grouping.

A. ab + 7b + 8a + 56

B. x2 + 3x – 2x - 6

C. 20x2 – 16x – 15x + 12

Trinomials in the form x2 + bx + c, where the leading coefficient is 1, can be factored by finding two numbers with a product of *c* and a sum of *b*. This can be written in the factored form as (x + p)(x + q) where pq = c and p + q = b.

If a trinomial is not factorable, it is considered to be prime.

**Objective 3:** Factor a trinomial where the leading coefficient is 1.

1. Write the trinomial in descending order of degrees.

2. List all the factors of *c*.

3. Find *p* and *q*, where the product of *p* and *q* is *c* and the sum of *p* and *q* is *b*.

4. Write the factored expression as (x + p(x + q).

5. Check your work by using the FOIL method.

Example #3: Factor the trinomial.

A. x2 + 2x - 15

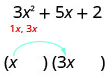
B. x2 – 7x + 6

C. 2x2 – 16x + 30

When a trinomial is written in the form ax2 + bx + c, you can use a couple of methods. The trial and error method begins with factoring the first term, then the last term. You must check the outside and inside to determine if the middle term is correct.

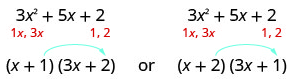
Factor the trinomial 3x2 + 5x + 2 using the “trial and error” method.

Find the “F” (first) factors. There is only



one way to write 3x2, which is x and 3x.

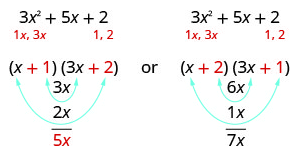
Find the “L” (last) factors. There is only one way



to write 2, which is 1 and 2. But, you need to

determine where to place the 1 and the 2.

To determine where the placement of the 1 and 2 go, you need to FOIL and check the “OI” (outer and inner) term.



Since the middle term is 5x, then the correct factorization of 3x2 + 5x + 2 is (x + 1)(3x + 2). This can also be written as (3x + 2)(x + 1) since multiplication is commutative.

**Objective 4a:** Factor a trinomial in the form ax2 + bx + c using the trial and error method.

1. Write the trinomial in descending order of degrees.

2. Factor any GCF.

3. Find all the factor pairs of the first term.

4. Find all of the factor parts of the third term.

5. Test all the possible combinations of the factors until the correct product is found.

6. Check using FOIL.

Example #4a: Factor the trinomial using the trial and error method.

A. 3y2 + 22y + 7

B. 2a2 - a – 3

C. 6b2 – 13b + 5

D. 15n3 – 85n2 + 100n

Another way of factoring trinomials in the form of ax2 + bx + c is by using the “ac” method. This is also known as the grouping method. You find two numbers with a product of ac and a sum of b. Then you will use the numbers to divide the x term into the sum of two terms and factor each portion of the expression separately. Lastly, you will factor out the GCF of the entire expression.

Factor the trinomial 6x2 + 7x + 2 using the “ac” method.

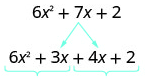


Find the product of a \* c.



Find two numbers that multiply to 12 and add to 7.

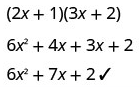
Both numbers need to be positive.



Rewrite 7x as 3x + 4x.



Factor by grouping.



Check your work by using FOIL.

**Objective 4b:** Factor a trinomial in the form ax2 + bx + c using “ac” method.

1. Factor out the GCF.

2. Find the product *ac*.

3. Find two numbers *m* and *n* that

Multiply to *ac*: *m* \* *n* = *a* \* *c*

Add to *b*: *m* + *n* = *b*

4. Split the middle term using m and n: *a*x2 + *m*x + *n*x + *c*

5. Factor by grouping.

6. Check by multiplying the factors or using the FOIL method.

Example #4b: Factor the trinomial using the “ac” method.

A. 10y2 – 55y + 70

B. 5x2 + 7x – 6

C. 2n2 – 27n – 45

A perfect square trinomial is a trinomial that can be written as the square of a binomial. Recall that when a binomial is squared, the result is the square of the first term added to twice the product of the two terms and the square of the last term. This can be used to factor any perfect square trinomial.

a2 + 2ab + b2 = (a + b)2

a2 – 2ab + b2 = (a – b)2

**Objective 5:** Factoring a perfect square trinomial.

1. Confirm that the first two and the last two terms are perfect squares.

2. Confirm that the middle term is twice the product of *ab*.

3. Write the factored form as (a + b)2 or (a - b)2.

Example #5: Factor the perfect square trinomials.

A. 20x2 + 20x + 4

B. 49x2 - 14x + 1

A difference of squares is a perfect square subtracted from a perfect square. Recall that the difference of squares can be rewritten as factors containing the same terms but opposite signs. This is because the middle terms cancel each out when the two factors are multiplied. You cannot factor the sum of squares.

a2 – b2 = (a + b)(a – b)

**Objective 6:** Factoring the difference of squares.

1. Confirm that there are only two terms.

2. Confirm that the first two and the last two terms are perfect squares.

3. Make sure there is a “-“ sign between the two terms.

4. Write the factored form as (a + b)(a – b).

Example #6: Factor the perfect square trinomials.

A. 9x2 - 25

B. 81y2 – 16

The sum and difference of cubes can be factored into a binomial and a trinomial, but with different signs.

a3 + b3 = (a + b)(a2 – ab + b2)

a3 - b3 = (a - b)(a2 + ab + b2)

An acronym that can be used to remember the signs when factoring the sum or difference of cubes is SOAP. The first letter of each word relates to the signs: Same Opposite Always Positive.

Using the binomial x3 – 8, break this down into x3 – 23 where a = x and b = 2 before factoring. The factors will be (x – 2)(x2 + 2x + 4). The sign of the first 2 is the *same* as the sign between x3 – 23. The sign of the 2x term is the *opposite* of the sign between x3 – 23 and the sign of the last term, 4, is *always positive*.

**Objective 7:** Factoring the sum or difference of cubes.

1. Confirm that the first and last terms are cubes.

2. For a sum of cubes, write the factored form as a3 + b3 = (a + b)(a2 – ab + b2). For a difference of cubes, write the factored form as a3 - b3 = (a - b)(a2 + ab + b2).

Example #7: Factor the sum or difference of cubes.

A. x3 - 512

B. 216 x3 + y3

Here is a summary of how to factor binomials, trinomials and polynomials. Always take out the GCF before proceeding with a method of factoring.

* Binomials
  + Differences of Squares: a2 – b2 = (a – b)(a + b)
  + Sum of Squares: Does not factor
  + Sum of Cubes: a3 + b3 = (a + b)(a2 – ab + b2)
  + Difference of cubes: a3 - b3 = (a - b)(a2 + ab + b2)
* Trinomials
  + x2 + bx + c: Trial and Error
  + ax2 + bx + c: Trial and Error or “ac” method
  + Perfect Squares: (a + b)2 = a2 + 2ab + c2
* Polynomials
  + Grouping

Review Problems: Factor by any method you prefer. Use the summary above as a guideline.

1. 4a2 – 12ab + 9b2

2. 24x3 + 81y3

3. 24y2 - 150

4. 40x2y + 44xy – 24y

5. 12x3y2 + 75xy2

6. 4x2 + 8bx – 4ax – 8ab

7. 7x3 – 21x2 - 70x

8. 6c2 + 41c + 63

9. 25p2 – 120p + 144

10. 9x2 – 12xy + 4y2 – 49

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OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.