Math 1111 – More on Functions and Their Graphs

Objectives:

1. Identify intervals where a function increases, decreases, or is constant

2. Use graphs to locate relative maxima or minima

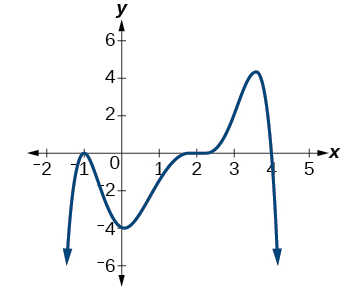
3. Test for symmetry

4. Identify even or odd functions and recognize their symmetries

As we begin to explore how functions change, we can identify intervals over which the function is changing in specific ways. Some of these specifics are where the graphs increase, decrease, or stay constant, where are the maximum and minimum points, and are the graphs symmetric.

**Objective 1:** Identify intervals where a function increases, decreases, or is constant.

When a function is increasing on an interval that means that the input values increase within that interval. Similarly, when a function is decreasing on an interval that means that the input values decrease within that interval. When a function is constant, that means that the input values stay the same within that interval.



Increasing 🡪

Increasing 🡪

🡨 Decreasing

Decreasing 🡪

🡨Increasing

Constant 🡪

The interval notation is: Increasing: (-∞, -1), Decreasing (-1, 0), Increasing (0, 1.5), Constant (1.5, 2.5), Increasing (2.5, 3.5), Decreasing (3.5, ∞)

This can also be written as: Increasing: (-∞, -1) U (0, 1.5) U (2.5, 3.5); Decreasing: (-1, 0) U (3.5, ∞); Constant: (1.5, 2.5).

Increasing, Decreasing and Constant Functions:

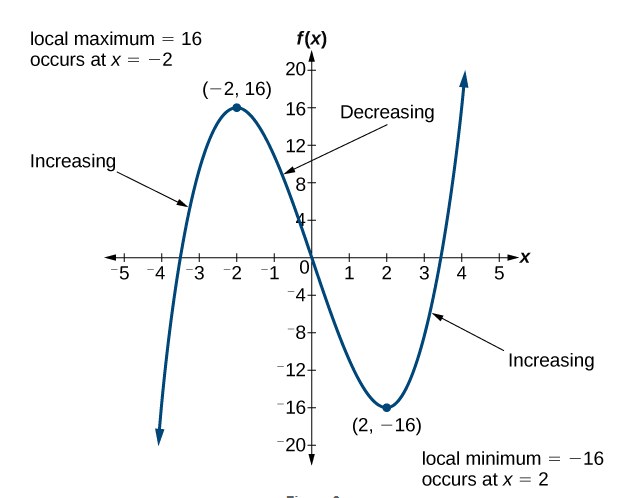
A function *f* is an increasing function on an open interval if *f (b)* > *f (a)* for any two input values *a* and *b* in the given interval where *b* > *a*. As the graph moves from the left to the right, the graph will rise.

A function *f* is a decreasing function on an open interval if *f (b)* < *f (a)* for any two input values *a* and *b* in the given interval where *b* < *a*. As the graph moves from the left to the right, the graph will fall.

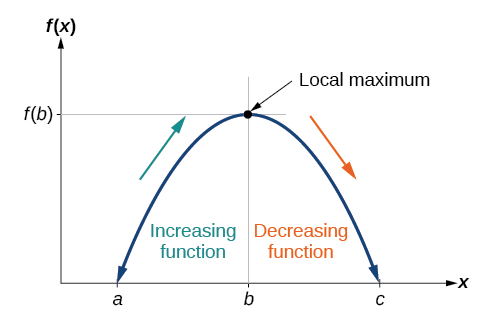
A function *f* is a constant function on an open interval if *f (b)* = *f (a)* for any two input values *a* and *b* in the given interval where *b* = *a*. As the graph moves from the left to the right, the graph will be a horizontal line.

**Objective 2:** Use graphs to locate relative maxima or minima.

When we are looking at a graph, we can locate the relative maximum (plural: maxima) or relative minimum (plural: minima) since the point is either higher or lower than the points directly beside it on both sides.

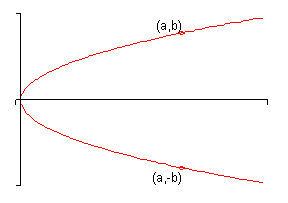


To locate the local maxima and minima from a graph, we need to observe the graph to determine where the graph attains its highest and lowest points, respectively, within an open interval. Like the summit of a roller coaster, the graph of a function is higher at a local maximum than at nearby points on both sides. The graph will also be lower at a local minimum than at neighboring points. The figure below illustrates these ideas for a local maximum.

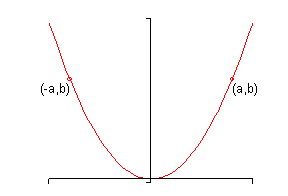


**Objective 3:** Tests for symmetry.

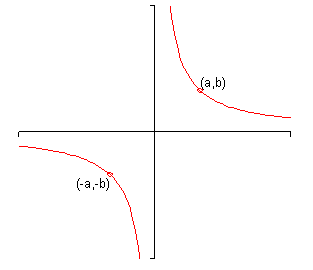
The Greek word for symmetry means “the same measure”. This means that the shape remains the same when you move it around the axis. The shape of a heart is symmetric. Notice that when you draw a line down the middle, the shape is the same on the right and left of the line.



A graph is considered symmetric to the x-axis when (a, b) is on the graph and so is (a, -b). Note: this is not a function because it does not pass the vertical line test.



A graph is considered symmetric to the y-axis when (a, b) is on the graph as so is (-a, b).



A graph is considered symmetric to the origin when (a, b) is on the graph as so is (-a, -b).

Here is how we test for symmetry.

|  |  |
| --- | --- |
| x-axis | Replace every y with –y and the result is the same as the original equation |
| y-axis | Replace every x with –x and the result is the same as the original equation |
| Origin | Replace every x and y with –x and –y and the result is the same as the original equation |

Example #3: Determine if the graphs are symmetric with respect to the x-axis, y-axis, or origin.

A. y = x2 – 6x4 + 2

B. y = 2x3 – x5

C. y4 + x3 – 5x = 0

D. x2 + y2 = 1

**Objective 4:** Identify even or odd functions and recognize their symmetries.

When testing for symmetry, we can determine if a function is even or odd. When a function is symmetric to the y-axis, it is considered an even function. When a function is symmetric to the origin, it is considered to be an odd function. When a graph is symmetric to the x-axis, it is neither an even or odd function because it is not a function.

Another way to determine if a function is even or odd is by looking at the exponents of the equation. If the equation has all odd exponents, then it is considered an odd function. If the equation has all even exponents, then it is considered an even function. If the equation has a combination of even and odd exponents, then it is considered being neither an even or odd function. If an equation has a constant, that can be written as x0, which would be considered an even exponent.

Example #4: Determine if the functions are odd, even, or neither.

A. f(x) = x5 – 3x

B. f(x) = 2x3 – 7

C. f(x) = 2x4 + 3x2 – 5

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.