Math 1111 – Polynomial and Rational Inequalities

Objectives:

1. Solve polynomial inequalities

2. Solve rational inequalities

**Objective 1:** Solve polynomial inequalities

A polynomial inequality is where a polynomial function, f(x), can be put into one of the following forms:

*f(x)* > 0, *f(x)* < 0, *f(x)* ≥ 0, *f(x)* ≤ 0

How to solve a linear inequality:

1. Get a zero on one side of the inequality.

2. Solve the equation *f(x)* = 0 by factoring or using the quadratic formula. These are the boundary points.

3. Graph the boundary points on a number line so that you have divided the number line into intervals.

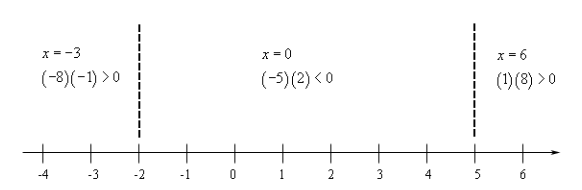
4. Choose test points within each of the intervals. Plug in the test points into the factored form of the polynomial. Since you are only looking to see if the answer is positive or negative, there is no reason to solve the problem.

5. Write down the answer. Look at the number line. If the test point from a region satisfies the inequality then that region is part of the solution. If the test point doesn’t satisfy the inequality then that region isn’t part of the solution.

For example: Solve x2 – 10 < 3x

Step #1: Rewrite as x2 – 3x – 10 < 0

Step #2: Factor: (x – 5)(x + 2) < 0 where x = 5 and x = -2 are the boundary points.

Step #3 - 4: Divide the number line into intervals at -2 and 5. Choose test points within the intervals.

Step #5: Find the answer. Since the only interval that satisfies this inequality is in the middle, then the answer is -2 < x < 5 or (-2, 5).

You can also use what you know about polynomial functions and their graphs to do steps 4 and 5. Since this is a 2nd degree polynomial, the shape of the graph is a parabola. Once you’ve divided your number line into the intervals separated by the boundary points of -2 and 5, determine the end behavior of the polynomial. Since the degree of 2 is even and the leading coefficient of 1 is positive, both ends are going up, or rising, which puts them above the x axis. Lastly, since the multiplicities at both zeros are 1, the graph must cross through the x axis at those points, leaving the middle of the parabola to be under the x axis, and thus the interval that satisfies the original inequality is (-2, 5).

Example #1: Solve the polynomial inequalities using the steps above. Write you answer in set-builder and interval notation.

A. x4 + 4x3 – 12x2 ≥ 0

B. (x + 1)(x – 3)2 > 0

C. 3x2 – 2x < 11

**Objective 2:** Solve rational inequalities

A rational inequality is where a rational function, f(x), can be put into one of the following forms:

*f(x)* > 0, *f(x)* < 0, *f(x)* ≥ 0, *f(x)* ≤ 0

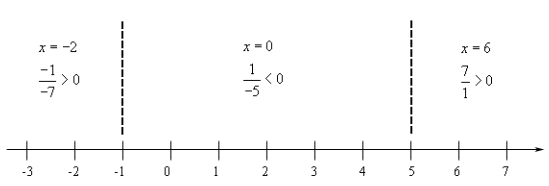
A rational inequality can be solved almost identical to a polynomial function, with a few minor differences. We do not solve rational inequalities like rational expressions where we clear the fractions. So, we just leave it in the rational form.

For example: Solve ≤ 0

Step #1: The inequality is already in the correct form.

Step #2: The inequality is already factored. We need to determine the boundary points which will come from making the numerator and denominator zero. This will be at x = -1 and x = 5.

Step #3 - 4: Divide the number line into intervals at -1 and 5. Choose test points within the intervals.



Step #5: Find the answer. So, we need the intervals that make the rational expression negative. That means the middle region. Also, since we’ve got an “or equal to” part in the inequality we also need to include where the inequality is zero, so this means we include x = −1. Notice that we will also need to avoid x = 5 since that gives division by zero. Therefore, the answer is -1 ≤ x < 5 or

[-1, 5).

Example #2: Solve the rational inequalities using the steps above. Write you answer in set-builder and interval notation.

A. > 0

B. < 0

C. ≥ 1 (HINT: You will have to move the 1 to the left side and find a common denominator to make a single rational expression.)

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.