Math 1111 – Quadratic Functions

Objectives:

1. Recognize parabola characteristics

2. Graph parabolas

3. Determine maximum and minimum values

4. Applications involving maximum and minimum values

Curved antennas, such as a satellite dish, are commonly used to focus microwaves and radio waves to transmit television and telephone signals, as well as satellite and spacecraft communication. The cross-section of the antenna is in the shape of a parabola, which can be described by a quadratic function.

When we are wanting to model area and projectile motion, it is easier to work with quadratic functions.

**Objective 1:** Recognize parabola characteristics

These are the characteristics of a quadratic function:

1. The greatest exponent of the equation is 2

2. The graph is a U-shaped curve called a parabola

a. If *a* (leading coefficient) > 0 the parabola opens upward

b. If *a* (leading coefficient) < 0, the parabola opens downward

3. The vertex is known as the turning point

a. If the parabola opens upward then the vertex is the minimum point

b. If the parabola opens downward then the vertex is the maximum point

4. A parabola has an axis of symmetry and is written in the form of a vertical line

a. It divides the parabola into equal halves

b. It goes through the vertex

5. Intercepts

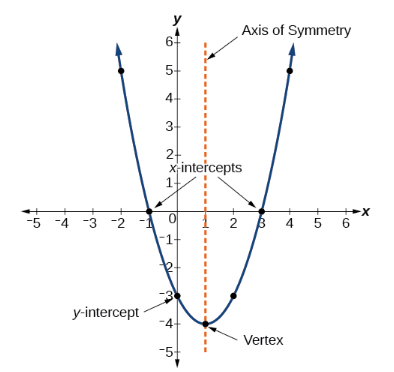
a. The y-intercept is the point at which the parabola crosses the y-axis.

b. The x-intercepts are the points at which the parabola crosses the x-axis. If they exist, the x-

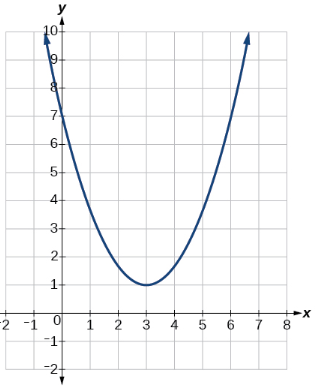
intercepts represent the zeros**,**or roots, of the quadratic function, the values of *x* at which *y* = 0.

These features are illustrated in the figure below for the quadratic function f(x) = (x + 1)2 – 4 or

f(x) = x2 – 2x - 3.



Example #1: Determine the vertex, axis of symmetry, zeros, and y-intercept of the parabola shown in the figure.



Vertex:

Axis of symmetry (write as a line):

Zeros (write as coordinate pairs):

y-intercept (write as coordinate pairs):

Does this have a maximum or minimum point?

**Objective 2:** Graph parabolas

Graphing quadratic functions in the general form *f*(x) = ax2 + bx + c

1. Determine if the parabola opens upward or downward.

2. Draw your axis of symmetry, . Since this is the x-value of the vertex, use this value to find the y-value of the vertex .

3. Find the x-intercepts (only the real solutions) by solving *f*(x) = 0. This can be done by factoring or using the quadratic formula.

4. Find the y-intercept by computing *f*(0).

5. Plot the vertex, axis of symmetry, intercepts, and any additional points (if needed). Connect the graph with a smooth curve.

Graphing quadratic functions in the standard form *f*(x) = a(x – h)2 + k

1. Determine if the parabola opens upward or downward.

2. Draw your axis of symmetry, x = h. Find the vertex (h, k).

3. Find the x-intercepts (only the real solutions) by solving *f(*x) = 0. This can be done by using the square root property.

4. Find the y-intercept by computing *f*(0).

5. Plot the vertex, axis of symmetry, intercepts, and any additional points (if needed). Connect the graph with a smooth curve.

Example #2: Using the steps above, draw the graphs of the given equations. Determine if there is a maximum or minimum point.

A. *f*(x) = x2 + 4x – 5.



B. *f*(x) = -x2 + 2x + 1



C. *f*(x) = (x + 3)2 - 4



D. *f*(x) = -2(x + 4)2 – 3



E. Write the equation of the parabola, in standard form, given that the vertex is (2, 4) and passes through the point (4, 16)

**Objective 3:** Determine maximum and minimum values

The output of the quadratic function at the vertex is the maximum or minimum value of the function, depending on the orientation of the parabola.

Example #3: Using the graphs in example #2 above, state the minimum or maximum point.

A.

B.

C.

D.

**Objective 4:** Applications involving maximum and minimum values

There are many real-world scenarios that involve finding the maximum or minimum value of a quadratic function, such as applications involving area and revenue.

Example #4: Solve each application problem.

A. A backyard farmer wants to enclose a rectangular space for a new garden within her fenced backyard. She has purchased 80 feet of wire fencing to enclose three sides, and she will use a section of the backyard fence as the fourth side.

1. Find a formula for the area enclosed by the fence if the sides of fencing perpendicular to the existing fence have length *L*.

2. What dimensions should she make her garden to maximize the enclosed area?

B. The unit price of an item affects its supply and demand. That is, if the unit price goes up, the demand for the item will usually decrease. For example, a local newspaper currently has 84,000 subscribers at a quarterly charge of $30. Market research has suggested that if the owners raise the price to $32, they would lose 5,000 subscribers. Assuming that subscriptions are linearly related to the price, what price should the newspaper charge for a quarterly subscription to maximize their revenue?

C. A ball is thrown upward from the top if a 40-foot high building at a speed of 80 feet per second. The ball’s height above ground can be modeled by the equation H(t) = -16t2 + 80t + 40.

1. When does the ball reach the maximum height?

2. What is the maximum height of the ball?

3. When does the ball hit the ground?

4. Sketch the path of the ball.

Technology: Follow the steps to check your work.

* Enter the quadratic equation
* Graph
* Adjust the windows, if needed
* 2nd TRACE, Maximum
* Locate left bound, ENTER, right bound, ENTER, ENTER (This gives your maximum)
* 2nd TRACE, Zero
* Locate left bound, ENTER, right bound, ENTER, ENTER (This gives you the x-intercept)

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.