Math 1111 – Combining Functions

Objectives:

1. Find the domain of a function

2. Combine functions using the algebra of functions, specifying domains

3. Form composite functions

4. Determine domains for composite function

5. Decompose functions

**Objective 1:** Find the domain of a function

The domain of function *f* is the largest set of real numbers for which the value of f(*x*) is a real number.

Consider the function f(x) = .

Since division by 0 is undefined, the denominator, x - 5, cannot be 0. Thus, x cannot equal 5. The domain of the function f(x),consists of all real numbers except 5. To represent this in interval notation, we state that the domain of f is ( -∞,5 ) U (5, ∞ ).

Now, consider a function involving a square root g(*x*) = .

Because only nonnegative numbers have square roots that are real numbers, the expression under the square root sign, x - 2, must be nonnegative. Therefore we can say, x – 2 ≥ 0. Solving for x we get

x ≥ 2. The domain of the function g(x) is the set of all real numbers that are greater than or equal to 2. To represent this in interval notation, we state the domain of g is {2, ∞).

Example #1: Find the domain of the following functions and write in interval notation.

A. t(x) = 2x – 3

B. f(x) = x2 – 7x

C. g(x) =

D. h(x) =

E. k(x) =

**Objective 2:** Combine functions using the algebra of functions, specifying domains

The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The **sum** of *f + g*, the **difference** *f - g*, the **product** *f \* g*, and the **quotient** are functions who domains are the set of all real numbers common to the domains of *f* and *g* (*Df Dg*), defined as follows:

1. Sum (*f + g*)(x) = *f(x)* + *g(x)*

2. Difference (*f - g*)(x) = *f(x)* - *g(x)*

3. Product (*f \* g*)(x) = *f(x)* \* *g(x)*

4. Quotient , provided *g(x)* ≠ 0

Example #2: Let f(x) = 3x - 1 and g(x) = x2 + x – 6, find each of the following functions. Then determine the domain of each function.

A. (f + g)(x)

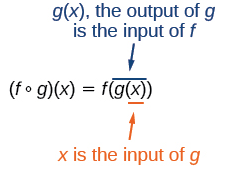
B. (f - g)(x)

C. (f \* g)(x)

D. (

**Objective 3:** Form composite functions

Besides performing algebraic operations on functions and combining them into a new function, we can also create functions by composing functions. When we wanted to compute a heating cost from a day of the year, we created a new function that takes a day as input and yields a cost as the output. The process of combining functions so that the output of one function becomes the input of another is known as a composition of function. We represent this as (f ∘ g) (x) = f(g(x)) and the left side is read as “*f* is composed with *g* at *x*” and the right side is read as “*g* of *g* of *x*.” The open circle symbol “∘” is the composite operator.



It is important to understand the order of operations in evaluating a composite function. We follow the usual convention with parentheses by starting with the innermost parentheses first, and then working to the outside.

In general, (f ∘ g) (x) ≠ (g ∘ f) (x) as shown in the example below.

Let f(x) = x2 and g(x) = x + 2, then

f(g(x)) = f(x + 2) g(f(x)) = g(x2)

= (x + 2)2 = x2 + 2

= x2 + 4x + 4

Example: Let’s try to interpret a composite function. Given the function *c(s)* gives the number of calories burned completing s sit-ups, and *s(t)* gives the number of sit-ups a person can complete in *t* minutes. Interpret *c(s(*3*))*.

Solution: The inside expression in the composite *s(*3*)*. Because the input to the *s*-function is time, t *= 3* represents 3 minutes, and *s*(3) is the number of sit-ups completed in 3 minutes

Using *s*(3) as the input to the function *c(s)* gives us the number of calories burned during the number of sit-ups that can be completed in 3 minutes or simply the number of calories burned in 3 minutes (by doing sit-ups).

Example #3A: Given f(x) = 2x + 1 and g(x) = x2 - 3x + 6, evaluate the composite functions.

A. (g ͦ f) (-3)

B. (f ͦ g) (7)

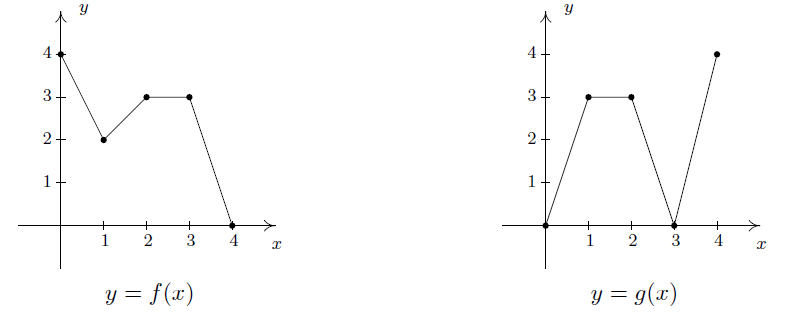
C. (f ͦ f) (-5)

D. (g ͦ g) (1)

Example #3B: Using a table to evaluate *f(g(3)*) and *g(f(3))*.

|  |  |  |
| --- | --- | --- |
| x | f(x) | g(x) |
| 1 | 6 | 3 |
| 2 | 8 | 5 |
| 3 | 3 | 2 |
| 4 | 1 | 7 |

Example #3C: Use the graphs of y = f(x) and y = g(x) below to find the function values.



A. (g ͦ f) (1)

B. (f ͦ g) (3)

C. (f ͦ g) (0)

D. (f ͦ f) (1)

**Objective 4:** Determine domains for composite functions

Given the function f(g(x)), determine the domain:

1. Find the domain of g.

2. Find the domain of f.

3. Find those inputs *x* in the domain of *g* for which *g(x)* is in the domain of *f*. That is, exclude those inputs *x* form the domain of *g* for which *g(x)* is not in the domain of *f*. The resulting set is the domain of (f ͦ g).

This needs to be done before we simplify.

Given: f(x) = 6x2, g(x) = x2 + 3x – 1, h(x) = 2x – 5, k(x) = , m(x) = , n(x) =

Example #4: Form the composite and find the domain.

A. (g ͦ f) (x)

B. (k ͦ h) (x)

C. (f ͦ n) (x)

D. (g ͦ m) (x)

**Objective 6:** Decompose functions

In some cases, it is necessary to decompose a complicated function. In other words, we can write it as a composition of two simpler functions. There may be more than one way to decompose a composite function, so we may choose the decomposition that appears to be most expedient.

Example #5: Given the following problems, write them as the composition of two functions.

a. f(x) =

b. | x2 + 7 |

c.

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