Math 1111 – Polynomial Functions and Their Graphs

Objectives:

1. Identify polynomial functions

2. Recognize characteristics of graphs of polynomial functions

3. Graph polynomial functions

a. Identify zeros and their multiplicities

b. Find the y-intercept

c. Determine the number of turning points

d. Determine end behavior

**Objective 1:** Identify polynomial functions

Let *n* be a non-negative integer. A polynomial function of degree *n* is a function that can be written in the form

*f(x) = anxn + an-1xn-1 + … + a2x2 + a1x + a0*

where each an is a coefficient and can be any real number except for zero and each *anxn* is a term of a polynomial function. The leading coefficient is the number an, the coefficient to the highest variable and the y-intercept of the graph is a0, which is the constant term.

Example #1: Determine if the following are polynomial functions. If so, find the degree, leading coefficient, and constant.

A. *f*(x) = 2x3 (3x + 4)

B. *g*(x) = -x3 + x5 – 2x + 3

C. *h*(x) = 3x5 (x + 4) (x – 1)

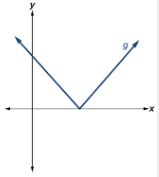
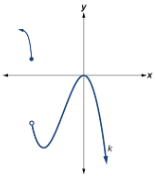
D. *m*(x) = – 5x + 2

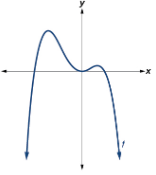
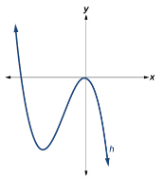
REVIEW: A polynomial that has a degree of zero is a function.

A polynomial that has a degree of one is a function.

A polynomial that has a degree of two is a function.

**Objective 2:** Recognize characteristics of graphs of polynomial functions

Polynomial Functions Not Polynomial Functions

Graphs of polynomials will always be nice smooth curves. There are no holes or breaks in the graph and there are no sharp corners in the graph.

The “humps” where the graph changes direction from increasing to decreasing or decreasing to increasing are often called **turning points**. If we know that the polynomial has degree *n* then we will know that there will be at most *n* − 1 turning points in the graph.

The zeroes of a polynomial are also the x­-intercepts of the graph. Recall that x-intercepts can either cross the x-axis or they can just touch the x-axis without actually crossing the axis.

Notice as well from the graphs above that the x-intercepts can either flatten out as they cross the x-axis or they can go through the x-axis at an angle. This is related to the multiplicity of the zeros.

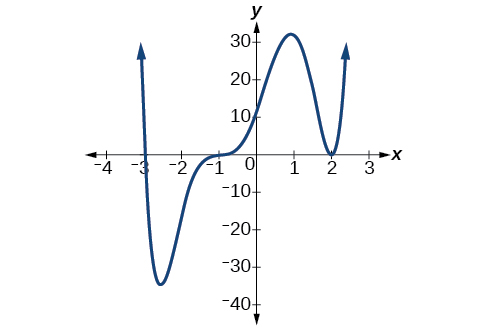
Finally, notice that as we let x get large in both the positive or negative sense (i.e. at either end of the graph) then the graph will either increase without bound or decrease without bound. This will always happen with every polynomial and it is described as the end behavior.

**Objective 3:** Graph polynomial functions

A. Identify zeros and their multiplicities: Graphs behave differently at various x-intercepts. Sometimes, the graph will cross over the horizontal axis at an intercept. Other times, the graph will touch the horizontal axis and "bounce" off.

For example, given the function f(x) = (x + 3) (x – 2)2(x + 1)3, notice that the behavior of the function at each of the x-intercepts is different.

(x + 3): The x-intercept at x = -3 cross the x-axis. The factor is linear and has a degree of one and an odd multiplicity.



(x – 2)2: The x-intercept at x = 2 bounces off the x-axis. The factor is quadratic and has a degree of two and an even multiplicity.

(x + 1)3: The x-intercept at x = -1 crosses but flattens out a bit first at the x-axis. The factor is cubic and has a degree of three and an odd multiplicity.

The x-intercepts of a polynomial function can be found by factoring. The zeros (also called roots) of a polynomial function are the x values for which f(x) = 0.

The Graphical Behavior of Polynomials at x-Intercepts

If a polynomial contains a factor of the form (x – *h*) p, the behavior near the x-intercept *h* is determined by the power *p*. We say that *x = h* is a zero and has a multiplicity *p*.

The graph of a polynomial function will touch the x-axis at zeros with even multiplicities. The graph will cross the x-axis at zeros with odd multiplicities.

The sum of the multiplicities is the degree of the polynomial.

Example #3a: Find the zeros and their multiplicities for the following functions. State whether the graph crosses or touches and turns at the x axis at each zero. Then find the degree of the polynomial function.

A. *f(x)* = x (x + 5)3(x + 1)2(x – 3)4

B. g(x) = x3 + 2x2 – 4x – 8

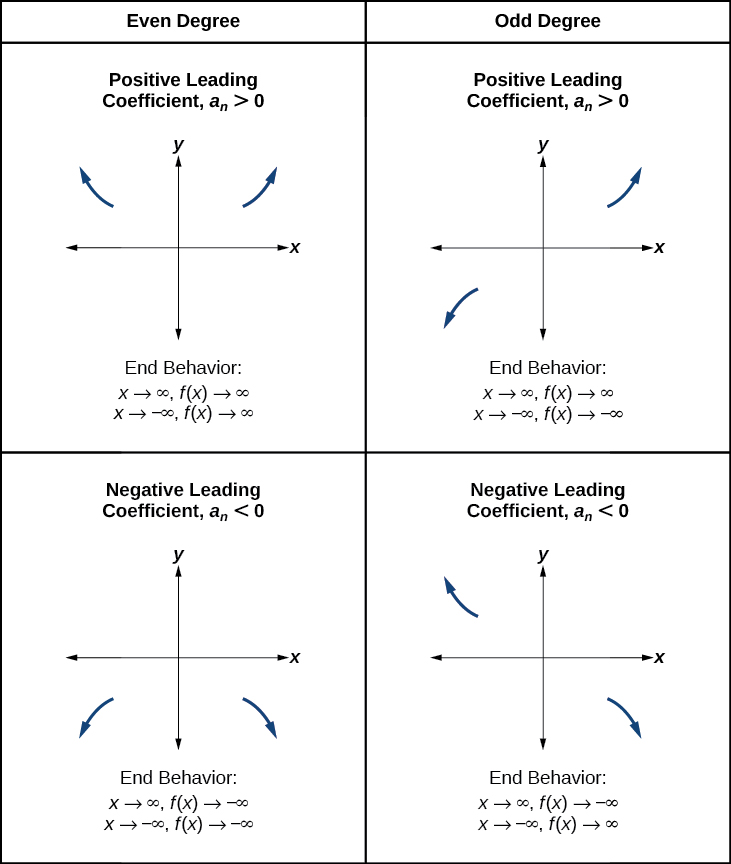
If a polynomial is in factored form, we need to evaluate f(0) to find the y-intercept. If the polynomial is in general form, then the constant term a0 is the y-intercept.

A turning point is a point of the graph where the graph changes from increasing to decreasing (rising to falling) or decreasing to increasing (falling to rising). A polynomial of degree *n* will have at most *n−1* turning points.

Example #3b: Find the y-intercepts and number of turning points of the polynomial function.

A. *f(x)* = -3(x - 5)2(x - 2)

B. g(x) = 5x4 + 4x3 – 10x – 16

The end behavior, which occurs at the far left and right of the graph of a polynomial function, is determined by the leading coefficient and the degree of the polynomial function. This is known as the Leading Coefficient Test.

The graph will either will either ultimately rise or fall as x increases without bound and will either rise or fall as x decreases without bound. This is because for very large inputs, say 100 or 1,000, the leading term dominates the size of the output. The same is true for very small inputs, say –100 or –1,000.

How to sketch a polynomial function?

1. Find the zeros of the polynomial function along with their multiplicities.

2. Find the y-intercept and the number of turning points.

3. Use the Leading Coefficient Test to determine the end behavior.

4. Determine if there is any symmetry: Recall that even functions have y axis symmetry and odd functions have origin symmetry.

5. Plot additional points to help with the sketch.

6. Use your graphing calculator to check your graph.

Example: Work through each of the steps and graph the polynomial functions.

1. f(x) = -2(x + 3)2(x – 5)



2. f(x) = ¼ x(x - 1)4(x + 3)3



3. f(x) = x4 - 2x3 - 3x2



4. f(x) = -x3 + x2 + 6x



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