Math 1111 – Dividing Polynomials; Remainder and Factor Theorems

Objectives:

1. Use long division

2. Use synthetic division

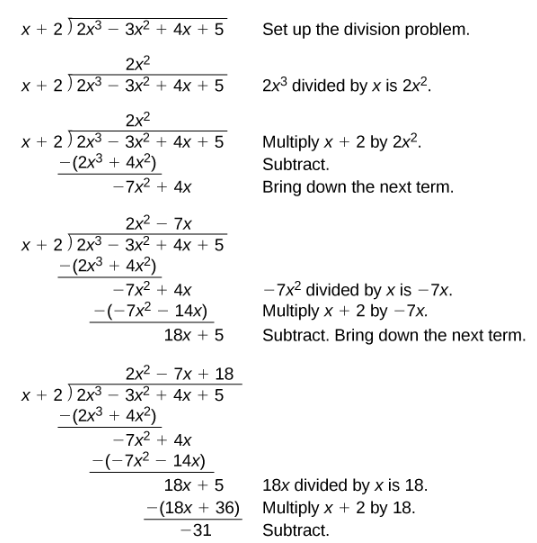
3. Evaluate a polynomial using the Remainder Theorem

4. Use the Factor Theorem to solve a polynomial equation

**Objective 1:** Use long division

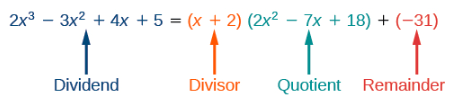
Division of polynomials that contain more than one term has similarities to long division of whole numbers. We can write a polynomial dividend as the product of the divisor and the quotient added to the remainder. The terms of the polynomial division correspond to the digits (and place values) of the whole number division. This method allows us to divide two polynomials.

For example, if we were to divide 2x3 − 3x2 + 4x + 5 by x + 2 using long division, it would look like this:



Therefore, = 2x2 – 7x + 18 - , or (x + 2)( 2x2 – 7x + 18) – 31.

Using this problem, we can identify the dividend, divisor, quotient, and remainder.



Since there is a remainder, we can conclude that x + 2 is not a factor of 2x3 − 3x2 + 4x + 5.

How to divide a polynomial by a binomial:

1. Set up the division problem.

2. Determine the first term of the quotient by dividing the leading term of the dividend by the leading term of the divisor.

3. Multiply the answer by the divisor and write it below the like terms of the dividend.

4. Subtract the bottom binomial from the top binomial.

5. Bring down the next term of the dividend.

6. Repeat steps 2–5 until reaching the last term of the dividend.

7. If the remainder is non-zero, express as a fraction using the divisor as the denominator.

Example #1: Divide each polynomial by the given binomial using long division.

A. 5x2 + 3x – 2 by x + 1

B. 6x3 + 11x2 – 31x + 15 by 3x - 2

**Objective 2:** Use synthetic division

As we’ve seen, long division of polynomials can involve many steps and be quite cumbersome. **Synthetic division** is a shorthand method of dividing polynomials for the special case of dividing by a linear factor whose leading coefficient is 1.

How to divide using synthetic division when the given divisor is in the form x – *k*:

1. Write *k* for the divisor.

2. Write the coefficients of the dividend. If there is a term missing, use a zero for the coefficient.

3. Bring the lead coefficient down.

4. Multiply the lead coefficient by *k*.  Write the product in the next column.

5. Add the terms of the second column.

6. Multiply the result by*k*.  Write the product in the next column.

7. Repeat steps 5 and 6 for the remaining columns.

8. Use the bottom numbers to write the quotient. The number in the last column is the remainder and has degree 0, the next number from the right has degree 1, the next number from the right has degree 2, and so on.

Example: Divide 5x2 - 3x - 36 by x - 3 using synthetic division.



Set up the synthetic division. Write *k* and the coefficients.

Bring down the lead coefficient. Multiply the lead coefficient by *k*. 

 Continue by adding the numbers in the second column. Multiply the resulting number by *k*. Write the result in the next column. Then add the numbers in the third column.

The result is 5x + 12. The remainder is zero so (x – 3) is a factor of the original polynomial.

Example #2: Divide each polynomial by the given binomial using synthetic division.

A. 3x4 + 18x3 – 2x + 40 by x + 7

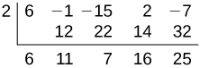
B. 4x3 + 10x2 – 6x - 20 by x + 2

**Objective 3:** Evaluate a polynomial using the Remainder Theorem

The Remainder Theorem states that if a polynomial f(x) is divide by x – *k*, then the remainder is the value f(*k*).

This can be used in place of evaluating a polynomial function.

Example: Use the Remainder Theorem to evaluate *f*(x) = 6x4 – x3 – 15x2 + 2x – 7 at x = 2.

 Using synthetic division, we find that the remainder is 25.

Therefore, *f*(2) = 25.

We can check our answer by evaluating *f*(2).

*f*(x) = 6x4 – x3 – 15x2 + 2x – 7

*f*(2) = 6(2)4 – (2)3 – 15(2)2 + 2(2) – 7

= 25

Example #3: Use the Remainder Theorem to evaluate 2x5 – 3x4 – 9x3 + 8x2 + 2 at x = -3

**Objective 4:** Use the Factor Theorem to solve a polynomial equation

The Factor Theorem states that *k* is a zero of *f(x)* if and only if (*x –* *k*) is a factor of *f(x)*.

How to use The Factor Theorem to factor a third-degree polynomial:

1. Use synthetic division to divide the polynomial by*(x − k)*.

2. Confirm that the remainder is 0.

3. Write the polynomial as the product of*(x − k)* and the quadratic quotient.

4. If possible, factor the quadratic.

5. Write the polynomial as the product of factors.

Example #4: Use the Factor Theorem to find the zeros of *f(x)* = x3 + 4x2 − 4x − 16 given that (x − 2) is a factor of the polynomial.

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.