Math 1111 – Exponential Functions

Objectives:

1. Evaluate exponential functions

2. Graph exponential functions

3. Evaluate functions with base *e*

4. Use compound interest formulas

**Objective 1:** Evaluate exponential functions

For any real number x, an exponential function is a function with the form f(x) = abx, where *a* is a non-zero real number called the initial value, and *b* is any positive real number such that *b* ≠1.

Evaluating Exponential Functions:

Recall that the base of an exponential function must be a positive real number other than 1. Why do we limit the base b to positive values? To ensure that the outputs will be real numbers. Observe what happens if the base is not positive:

Let b = −3 and x = . Then, f(x) = f() = =, which is not a real number.

Why do we limit the base to positive values other than 1? Because base 1 results in the constant function. Observe what happens if the base is 1:

Let b = 1.  Then f(x) = 1x = 1 for any value of x.

To evaluate an exponential function with the form f(x) = bx, we simply substitute x with the given value, and calculate the resulting power. For example:

Let f(x) = 2x. What is f (3)?

f (3) =23 = 8

To evaluate an exponential function with a form other than the basic form, it is important to follow the order of operations. For example:

Let f(x) = 30(2)x.  What is f(3)?

f(3) = 30(2)x Equation

f(3) = 30(2)3  Replace x with 3

f(3) = 30(8) Calculate the exponential

f(3) = 240 Multiply

If the order of operations were not followed, the results would be incorrect. For example:

f(3) = 30(2)x Equation

f(3) = 603 Multiplication

f(3) = 216,000 Calculate the exponential

Example #1: Evaluate the exponential functions

A. Let f(x) = 5(3)x + 1. Evaluate f(2) without using a calculator.

B. Let f(x) = 8(1.2)x+1. Evaluate f(3) using a calculator. Round to four decimal places.

**Objective 2:** Graph exponential functions

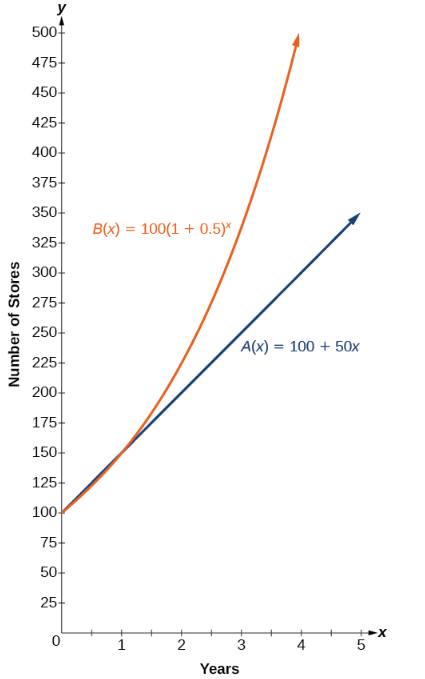
A function that models exponential growth grows by a rate proportional to the amount present. For any real number x and any positive real numbers a and b such b ≠ 1, an exponential growth function has the form f(x) = abx, where a is the initial or starting value of the function and b is the growth factor or growth multiplier per unit x.

To differentiate between linear and exponential functions, let’s consider two companies, A and B. Company A has 100 stores and expands by opening 50 new stores a year, so its growth can be represented by the function A(x) = 100 + 50x.  Company B has 100 stores and expands by increasing the number of stores by 50% each year, so its growth can be represented by the function

B(x) = 100 (1 + 0.5)x.

A few years of growth for these companies are illustrated below.

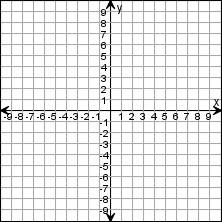
|  |  |  |
| --- | --- | --- |
| Year, *x* | Stores, Company A | Stores, Company B |
| 0 | 100 + 50(0) = 100 | 100 (1 + .05)0 = 100 |
| 1 | 100 + 50(1) = 150 | 100 (1 + .05)1 = 150 |
| 2 | 100 + 50(2) = 200 | 100 (1 + .05)2 = 225 |
| 3 | 100 + 50(3) = 250 | 100 (1 + .05)3 = 337.5 |
| *x* | *A(x)* = 100 + 50*x* | B(*x*) = 100 (1 + 0,5)*x* |

The graphs comparing the number of stores for each company over a five-year period are shown in the [Figure](https://cnx.org/contents/E6wQevFf@14.23:aU0u6kE1@21/Exponential-Functions#CNX_Precalc_Figure_04_01_001)**.** We can see that, with exponential growth, the number of stores increases much more rapidly than with linear growth.

Note that the domain for both functions is [0, ∞) and the range for both functions is [100, ∞). After year 1, Company B always has more stores than Company B.

Now we will turn our attention to the function representing the number of stores for Company B, B(x) = 100 (1 + 0.5)x. In this exponential function, 100 represents the initial number of stores, 0.50 represent the growth rate, and

1 + 0.5 = 1.5 represents the growth factor. Generalizing further, we can write this function as B(x) = 100(1.5)*x*, where 100 is the initial value, 1.5 is called the base, and *x* is called the exponent.

Example #2: Graph the following exponential functions.

A. Graph f(x) = 2x and answer the questions.

|  |  |
| --- | --- |
| x | f(x) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

1. Is the graph a function? Is the graph one-to-one?

How do you know?

2. Is the graph increasing or decreasing? Give the interval(s) where the function is increasing or decreasing.

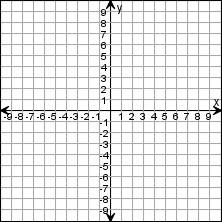
3. Are there any values of x that would make 2x undefined?

4. State the domain and range.

5. Give the intercepts, if there are any.

6. As the value of x gets large, what happens to the value of 2x? Similarly, as the value of x gets very small, what happens to the value of 2x?

7. Does the graph have any asymptotes? If yes, give the asymptote(s), written as a line.



B. Graph f(x) = ½x and answer the questions.

|  |  |
| --- | --- |
| x | f(x) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

1. Is the graph a function? Is the graph one-to-one?

How do you know?

2. Is the graph increasing or decreasing? Give the interval(s) where the function is increasing or decreasing.

3. Are there any values of x that would make 2x undefined?

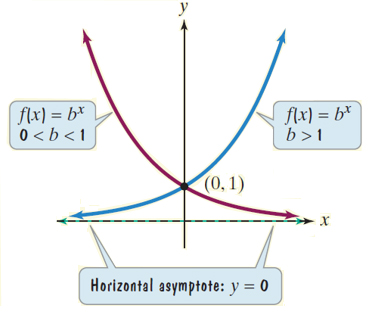
4. State the domain and range.

5. Give the intercepts, if there are any.

6. As the value of x gets large, what happens to the value of 2x? Similarly, as the value of x gets very small, what happens to the value of 2x?

7. Does the graph have any asymptotes? If yes, give the asymptote(s), written as a line.

Characteristics of Exponential Functions

 1. The domain of f(x) = bx consists of all real numbers;

(- ∞, ∞). The range of f(x) = bx consists of all positive real numbers; (0, ∞).

2. The graphs of all exponential functions of the form

f(x) = bx passes through the point (0, 1) because

f(0) = b0 = 1 (b ≠ 0). The y-intercept is (0, 1). There is no x-intercept.

3. If b > 1, f(x) = bx has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.

4. If 0 < b < 1, f(x) = bx has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.

5. The graph f(x) = bx is a one-to-one function and has an inverse that is a function.

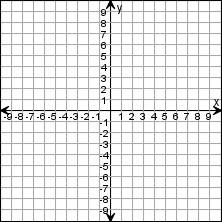
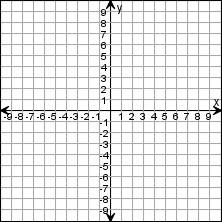
6. The graph of f(x) = bx approaches, but does not touch, the x-axis. The x-axis, or y = 0, is a horizontal asymptote.

The standard form of an exponential function is f(x) = ab(x – h) + k. The following tables describes the transformations.

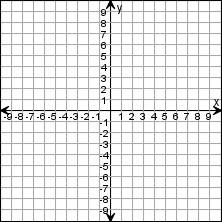
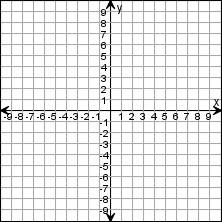
|  |  |
| --- | --- |
| Transformation | Description |
| Horizontal Shift | * If *h* > 0, then shift left *h* units * If *h* < 0, then shift right *h* units * The horizontal asymptote is at y = k |
| Vertical Stretching or Shrinking | Multiplying y-coordinates by *a*   * Stretches the graph if *a* > 1 * Shrinks the graph if 0 < *a* < 1 |
| Reflecting | * If *b* is negative, reflects around x-axis * If *x* is negative, reflects around the y-axis |
| Vertical Shift | * If *k* > 0, then shift upward *k* units * If *k* < 0, then shift downward *k* units |

Graph the following exponential functions using transformations.

C. f(x) = 2x + 2 D. f(x) = -2x



E. f(x) = 2x –2 + 3 F. f(x) = -2x - 1



**Objective 3**: Evaluate functions with base *e*

The number *e* represents the irrational number (1 + ) n, as n increases without bound. The approximate value of *e* to nine decimal places is *e* ≈ 2.718281827. This is also known as the natural base. The

f(x) = ex is called the natural exponential function. (To find the *e* button on your calculator 🡪 shift (or 2nd key) ln.

Example #3: Solve each problem.

A. The exponential function f(x) = 1145e0.0325x models the gray world population of the Western Lakes, f(x), x years after 1978. Project the gray wolf’s population in the recovery area in 2017.

B. In the year 2004, the initial population of a town is 150,000. The population is decreasing by 4.5% a month due to a poor economy. What is the population in the year 2011?

**Objective 4**: Use compound interest formulas

After *t* years, the balance, *A*, in an account with principal *P* and an annual interest rate *r* (in decimal form) is given by the following formulas:

For *n* compounding periods per year, where n can be monthly, quarterly, etc.

For continuous compounding: A = P*e*rt

Example #4: Solve each of the following problems.

A. A sum of $10,000 is invested at an annual rate of 6%. Find the balance in the account after 10 years subject to quarterly compounding.

B. A sum of $10,000 is invested at an annual rate of 5%. Find the balance in the account after 10 years subject to continuous compounding.

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.