Math 1111: Systems of Nonlinear Equations: Two Variables

Objectives:

1. Recognize systems of nonlinear equations in two variables

2. Solve systems of nonlinear equations by substitution

3. Solve systems of nonlinear equations by addition

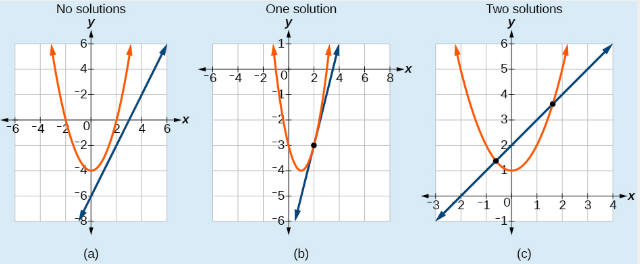
4. Applications using systems of nonlinear equations

A system of nonlinear equations is a system of two or more equations in two or more variables containing at least one equation that is not linear. Recall that a linear equation can take the form

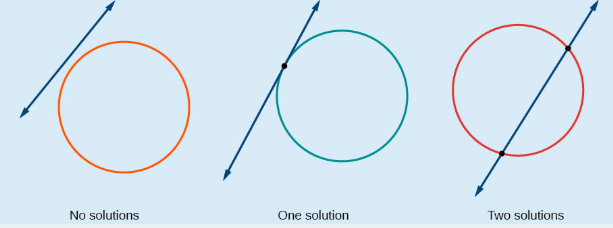
Ax + By + C = 0.  Any equation that cannot be written in this form in nonlinear. The substitution method we used for linear systems is the same method we will use for nonlinear systems. We solve one equation for one variable and then substitute the result into the second equation to solve for another variable, and so on. There is, however, a variation in the possible outcomes.

**Objective 1:** Recognize systems of nonlinear equations in two variables.

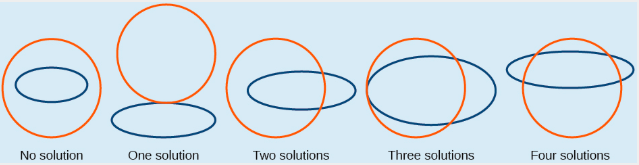
* A line and a parabola: Given an equation Ax + By + C = 0 and Ax2 + Bx + C = 0, there are three possible solutions.



* A line and a circle: Given an equation Ax + By + C = 0 and (x – h)2 + (y – k)2 = r2, there are three possible solutions.



* A circle and an ellipse: Given an equation (x – h)2 + (y – k)2 = r2 and ax2 + by2 + cx + dy + e = 0 there are five possible solutions.



**Objective 2:** Solve systems of nonlinear equations by substitution.

1. Solve the linear equation for one of the variables.

2. Substitute the expression obtained in step one into the parabola equation.

3. Solve for the remaining variable.

4. Check your solutions in both equations.

Example #1: Solve the following system of nonlinear equations by substitution.

A. x - y = -1

y = x2 + 1

B. x2 + y2 = 5

3x – y = 5

**Objective 3:** Solve systems of nonlinear equations by addition (basically the same method as solving systems of linear equations).

1. Write both equations with the x- and y-variables on the left side of the equal sign and constants on the right.

2. Write one equation above the other, lining up corresponding variables. If one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, add the equations together, eliminating one variable. If not, use multiplication by a nonzero number so that one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, then add the equations to eliminate the variable.

3. Solve the resulting equation for the remaining variable.

4. Substitute that value into one of the original equations and solve for the second variable.

5. Check the solution by substituting the values into the other equation.

Example #2: Solve the following system of nonlinear equations by substitution.

A. x2 + y2 = 26

3x2 + 25y2 = 100

B. 4x2 - 9y2 = 36

4x2 + 9y2 = 36

**Objective 4:** Solve application problems using systems of nonlinear equations.

Example #3: Solve the application problem.

A. Two numbers add up to 300. One number is twice the square of the other number. What are the numbers?

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