Math 1111 – Properties of Logarithms and Solving Exponential and Logarithmic Equations

Objectives:

1. Use the product, quotient and power rule to expand and condense logarithms

2. Use the change-of-base property

3. Use like bases to solve exponential equations

4. Use logarithms to solve exponential equations

5. Use the one to one property of logarithms to solve logarithmic equations

6. Use the definition of a logarithm to solve logarithmic equations

7. Solve applied problems involving exponential and logarithmic equations

**Objective 1:** Use the product, quotient and power rule

The Product Rule

The logarithm of a product is the sum of the logarithms: logb MN = logb M + logb N

The Quotient Rule

The logarithm of a quotient is the difference of the logarithms: logb MN = logb M - logb N

The Power Rule

The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number: logb Mp = p logb M

Example #1: Use the product, quotient, and/or power rule to expand or condense the logarithms.

A. log (1000x)

B. ln (

C. log5

D. log (50) + log (4)

**Objective 2:** Use the change-of-base property

When you have to calculate using a base other than 10, you can use the change-of-base property.

Example #2: Use the change-of-base property to evaluate log7 2506.

**Objective 3:** Use like bases to solve exponential equations

If bM = bN, then M = N.

Steps for solving exponentials with the same base.

1. Rewrite the equation in the form bM = bN

2. Set M = N

3. Solve for the variable

Example #3: Solve each of the equations using the same base.

A. 53x - 6 = 125

B. 42x - 1 =

C. 8x + 2 = 4x - 3

D. 8x + 3 = 16x - 1

**Objective 4:** Use logarithms to solve exponential equations

Sometimes the terms of the exponential equation cannot be rewritten with a common base, so you can take the logarithm of both sides of the equation.

Steps for solving an exponential equation using logarithms.

1. Isolate the exponential expression.

2. Take the common logarithm on both sides of the equation for base 10. Take the natural log on both sides of the equation for bases other than 10.

3. Simplify using one of the following properties:

a. ln bx = x ln b

b. ln ex

c. log 10x = x

4. Solve for the variable.

Example #4: Solve each of the equations using log or ln.

A. 5x = 124

B. 10x = 8000

C. Solve 10x = 3.91

D. 32x - 1 = 7x + 1

**Objective 5:** Use the one to one property of logarithms to solve logarithmic equations

As with exponential equations, we can use the one-to-one property to solve logarithmic equations.

For any real numbers x > 0, S > 0, T > 0 and any positive real number b, where b ≠ 1,

logb S = logb T if and only if S = T

For example, log2 (x – 1) = log2 8, then (x – 1) = 8.

Steps to using the one-to-one property to solve logarithmic equations.

1. Express the equation in the form = This form involves a single logarithm whose coefficient is 1 on each sides of the equation. Use the rules of logarithms to rewrite the sides, if applicable.

2. Use the one-on-one property to rewrite the equation without logarithms. If logb M = logb N, then M=N.

3. Solve for the variable.

4. Check proposed solution in the original equation. Include in the solution set only values for which

M > 0 and N > 0.

Example #5: Solve each of the equations.

A. log6 (3n + 5) = log6 (2n + 6)

B. log8 (2x + 5) = log8 (-3x + 10)

C. log (*x* + 4) − log 2 = log (5*x* +1)

D. ln (x - 3) = ln (7x - 23) – ln (x + 1)

**Objective 6:** Use the definition of a logarithm to solve logarithmic equations

Sometimes a logarithmic equation can only be solved if rewritten as an exponential equation.

For any algebraic expression S and real numbers b and c, where b > 0, b ≠ 1,

logb (x) = c if and only if bc = S

Steps to using the definition of a logarithm to solve equations.

1. Express the equation in the form

2. Use the definition of log to rewrite the equation in exponential form means bc = M.

3. Solve for the variable.

4. Check proposed solutions in the original equation. Include in the solution set only values for which M > 0.

Example #6: Solve each logarithmic equation.

A. Solve: log2 (x - 4) = 3

B. log4 (x + 5) = 3

C. 4 ln (3x) = 8

D. log x+ log (x - 3) = 1

E. log5 x + log5 (4x – 1) = 1

To solve these problems, you will have to use your algebra skills and logarithm and exponential properties.

1. 2 lnx + 3 = 7

2. 2 ln(6x) = 7

3. 4e2x + 5 = 12

4. e2x – ex = 56

**Objective 7:** Solve applied problems involving exponential and logarithmic equations

Examples: Solving using exponentials or logarithms. Show all of your work.

A. How long, to the nearest tenth of a year, will it take $1000 to grow to $3600 at 8% annual interest compounded quarterly?

B. How long, to the nearest tenth of a year, will it take $8000 to grow to $16,000 at 8% annual interest compounded continuously?

C. The formula *A* = 37.3*e0.0095t* models the population of California, *A*, in millions, *t* years after 2010. When will the population of California reach 40 million?

D. The percentage of adult height attained by a girl who is *x* years old can be modeled by *f*(*x*) = 62 + 35 log (*x −* 4) where *x* represents the girl’s age (from 5 to 15) and *f*(*x*) represents the percentage of her adult height. At what age has a girl attained 97% of her adult height?

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.