Math 1111 – Linear Inequalities and Absolute Value Inequalities

Objectives:

1. Use interval and set-builder notation

2. Solve linear inequalities

3. Recognize a no solution or infinite solution set

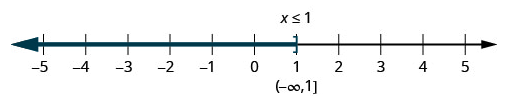
4. Solve compound inequalities

5. Solve absolute value inequalities

A linear inequality is a linear expression that uses an inequality sign.

**Objective 1:** Understanding interval and set-builder notation

Indicating the solution to an inequality such as x ≤ 1 can be achieved in several ways, such as by a number line, interval notation, and set-builder notation.



We can use this number line above that shows a black ray that begins at x = 1 and, as indicated by the arrowhead, continues to negative infinity. The bracket on the number line indicates that the 1 is inclusive in the solution set. This answer can be expressed as (-1] in interval notation and can be read as, “the solution set that is equal to 1 or less than 1”. This answer can also be expressed in set-builder notation as {x | x ≤ 1} which can be read as, “all real numbers x such that x is less than or equal to 1”.

The interval notation is probably the most used notation, especially in other higher-level math courses. The main concept to remember is that parentheses represent solutions greater or less than the number, and brackets represent solutions that are greater than or equal to or less than or equal to the number. Use parentheses to represent infinity or negative infinity since they are not numbers in the usual sense of the word and, therefore, cannot be “equaled”.

An example of an interval such as [-2, 6) is all of the numbers between -2 and 6 and including -2.

|  |  |  |  |
| --- | --- | --- | --- |
| Set Indicated | Set-Builder Notation | Interval Notation | Graph |
| All real numbers between a and b, but not including a or b | {x | a < x < b} | (a, b) |  |
| All real numbers greater than a, but not including a | {x | x > a} | (a, ) |  |
| All real numbers less than b, but not including b | {x | x < b} | ( b) |  |
| All real numbers greater than a, including a | {x | x ≥ a} | [a, ) |  |
| All real numbers less than b, including b | {x | x ≤ b} | ( b] |  |
| All real numbers between a and b, including a | {x | a ≤ x < b} | [a, b) |  |
| All real numbers between a and b, including b | {x | a < x ≤ b} | (a, b] |  |
| All real numbers between a and b, including a and b | {x | a ≤ x ≤ b} | [a, b] |  |
| All real numbers | {x | x is all real numbers} | (, ) |  |

Example #1: Use interval and set-builder notation to represent each of the questions.

A. Numbers between -2 and including 5.

B. All numbers greater than -2.

C. All numbers less and including 1.

D. All numbers greater than 7 and less than and including -3.

**Objective 2:** Solve linear inequalities.

When we work with inequalities we can usually treat them similarly to linear equations. We can use the addition and multiplication properties to help us solve them. But, when we multiply or divide by a negative numbers, the inequality sign must reverse.

Properties of Inequalities

Addition Property: If a < b, then a + c < b + c

Multiplication Property: If a < b and c > 0, then ab < bc

If a < b and c < 0, then ab > bc

These properties also apply to a ≤ b, a > b, a ≥ b

Example #2: Solve the following linear inequalities. Write your answer in interval and set-builder notation. Graph each answer.

A. x + 7 > 9

B. 6 ≥ x + 1

C. 3x – 2 < 1

D. -2x – 1 ≥ 5

E. 4x + 7 ≥ 2x – 3

F.

G.

**Objective 3:** Recognize a no solution or infinite solution set.

Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality if the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction. If the statement is true, then the solution set is all real numbers. If the statement is false, then the solution set is the empty set.

Example #3: Determine if the equations represent an identity or a contradiction.

A. 8x – 2(5 – x) < 4 (x + 9) + 6x

B. 4b – 3(3 – b) < 5 (b – 6) + 2b

C.

**Objective 4:** Solving compound inequalities.

A compound inequality involves two inequalities in one statement. For example if you have two inequalities such as:

5 < 2x + 3 and 2x + 3 < 9

you can put these together into a compound inequality:

5 < 2x + 3 < 9

This form allows you to solve one equation by performing the same operation on all three parts of the inequality and isolating the x (or another variable) in the middle.

Example #4: Solve the following linear inequalities. Write your answer in interval and set-builder notation. Graph each answer.

A. 3 ≤ 3x + 5 < 6

B. 4 < 2x + 8 ≤ 10

C. 3y < 4 – 5y < 5 + 2y

**Objective 5:** Solving absolute value inequalities.

The absolute value of a number is the distance between two points. Regardless of the direction, positive or negative, the distance between the two points is represented as a positive number or a zero.

To solve absolute value inequalities given the algebraic expression X, and *k* > 0, rewrite the inequalities in the form:

| X | < *k* is equivalent to - *k* < X < *k*

| X | > *k* is equivalent to X < - *k* or X > *k*

These statements also apply to | X | ≤ *k* and | X | ≥ *k*.

Example #5: Solve the following absolute value inequalities. Write your answer in interval and set-builder notation. Graph each answer.

A. | x – 1 | ≤ 3

B. - ½ | 4x – 5 | + 3 < 0

C. 5 | 2x – 1 | + 6 ≥ 10

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.