Math 1111: Transformations of Graphs

Objectives:

1. Recognize graphs of common functions

2. Use vertical shifts to graph functions

3. Use horizontal shifts to graph functions

4. Use reflections to graph functions

5. Use vertical stretching and shrinking to graph functions

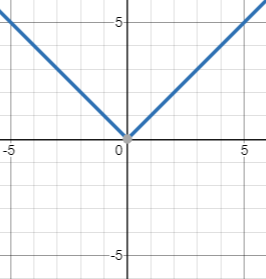
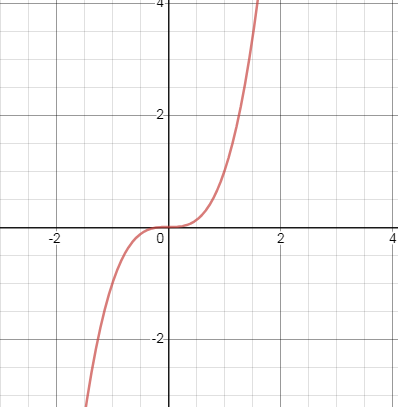
6. Use horizontal stretching and shrinking to graph functions

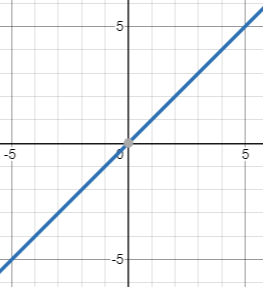
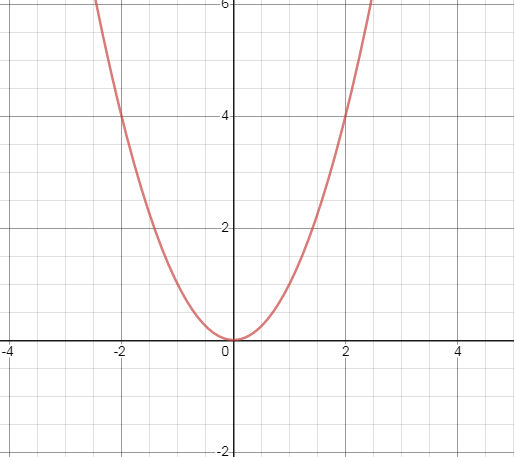
7. Graph functions involving a sequence of transformations

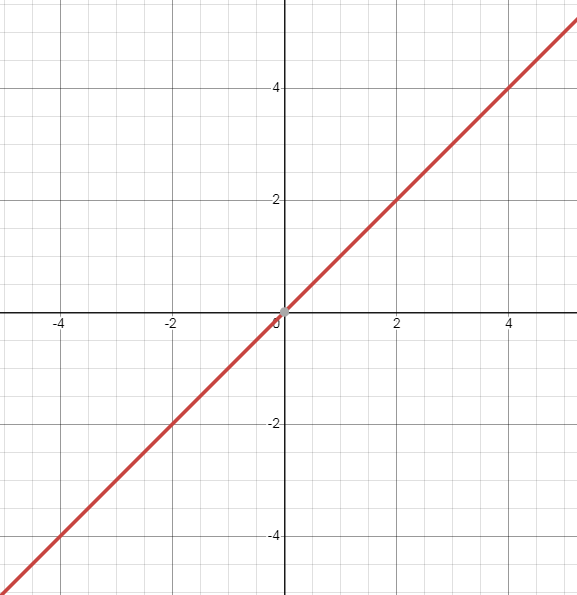
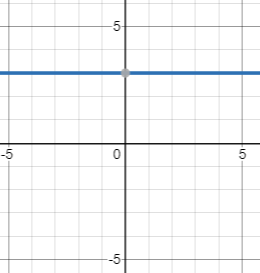
**Objective 1:** Recognize graphs of common functions

Below are some common graphs that you must be able to recognize.

Constant Function: f(x) = c Identity Function: f(x) = x Absolute Function: f(x) = |x|

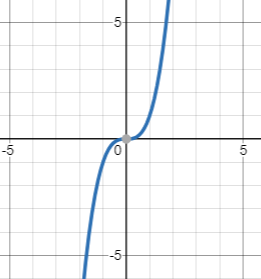


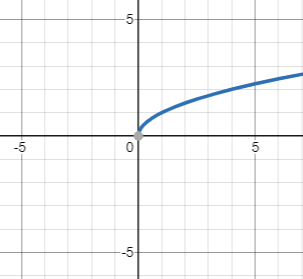
Domain is (-∞, ∞) Domain is (-∞, ∞) Domain is (-∞, ∞)

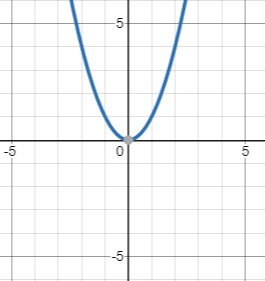
Range is c Range is (-∞, ∞) Range is [0, ∞)

Function is even Function is odd Function is even

Quadratic Function: f(x) = x2 Square Root Function: f(x) = Cubic Function: f(x) = x3





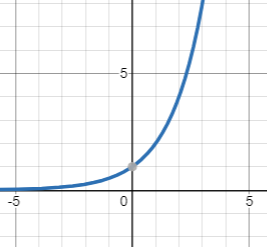


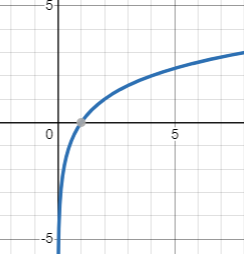
Domain is (-∞, ∞) Domain is [0, ∞) Domain is (-∞, ∞)

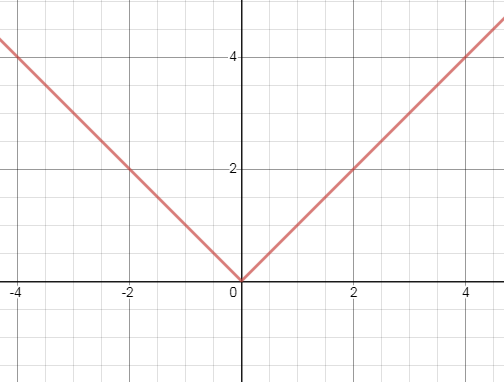
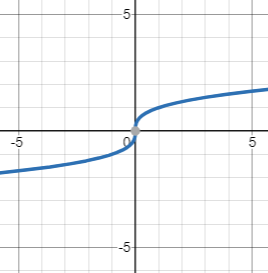
Range is [0, ∞) Range is [0, ∞) Range is (-∞, ∞)

Function is even Function is neither Function is odd

Cube Root Function: f(x) = Exponential Function: f(x) = bx Logarithmic Function: f(x) = logb(x)







Domain is (-∞, ∞) Domain is (-∞, ∞) Domain is (0, ∞)

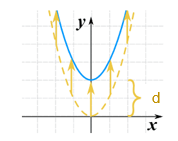
Range is (-∞, ∞) Range is (0, ∞) Range is (-∞, ∞)

Function is odd Function is neither Function is neither

Horizontal Asymptote: y = 0 Vertical Asymptote: x = 0

**Objective 2:** Use vertical shifts to graph functions.

When given a function f(x), a new function g(x) = f(x) + *d*, where *d* is a constant, there is a vertical shift of the function f(x). All of the output values change by *d* units. If *d* is positive then the graph will shift up *d* units and if *d* is negative the graph will shift down *d* units.

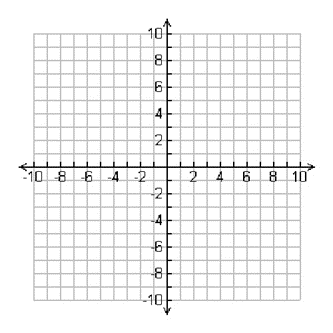
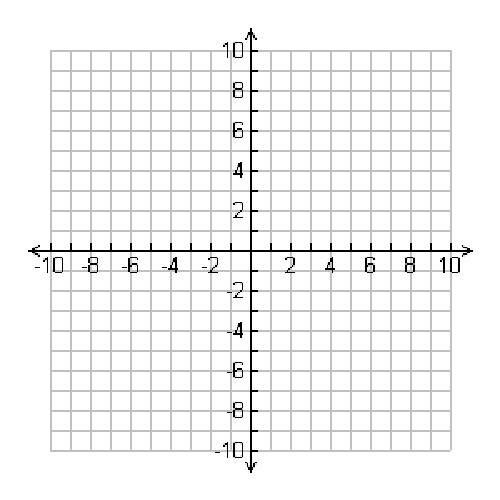
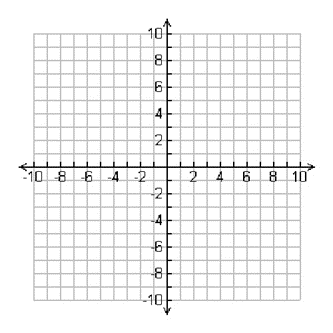


Given the quadratic function f(x) = x2, the new graph f(x) = x2 + 2

is a vertical shift by *d* = 2.

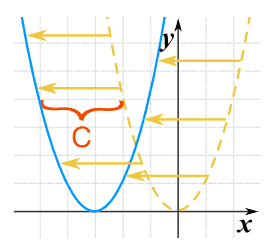
Example #2: Sketch the parent graph and then the given transformation of the parent graph.

A. f(x) = x2 + 3 B. g(x) = |x| - 4 C. h(x) = x3 - 2

**Objective 3:** Use horizontal shifts to graph functions.

When given a function f(x), a new function g(x) = (x – *c*), where *c* is a constant, there is a horizontal shift of the function f(x). If *c* is positive, then the graph will shift to the left *c* units and if *c* is negative, then the graph will shift to the right *c* units.

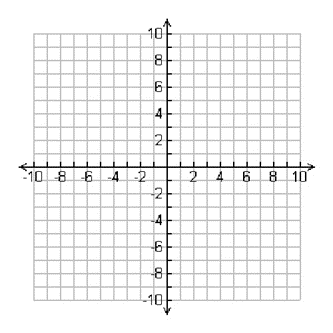
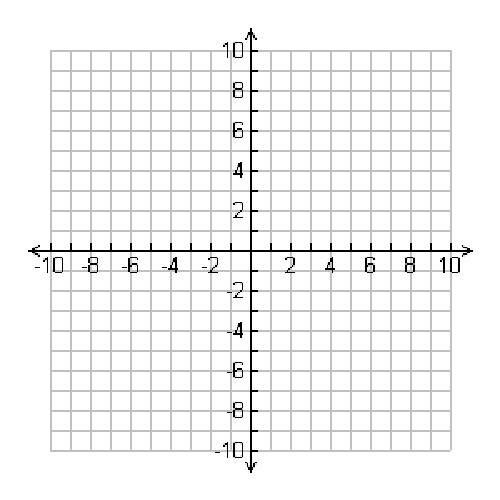
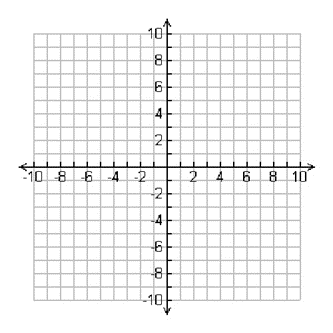


Given the quadratic function f(x) = x2, the new graph f(x) = (x + 3)2

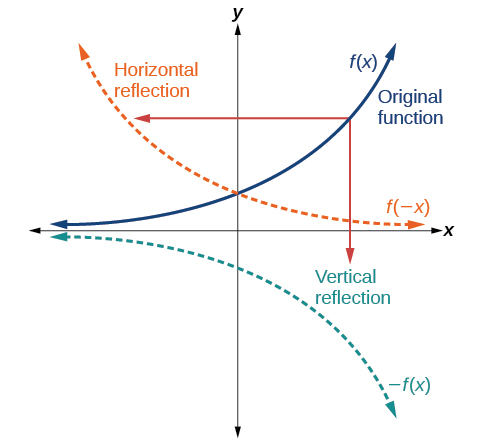
is a horizontal shift by *c* = -3.

Example #3: Sketch the parent graph and then the given transformation of the parent graph.

A. f(x) = B. g(x) = | x + 4 | C. h(x) = (x + 2)3

**Objective 4:** Use reflections to graph functions

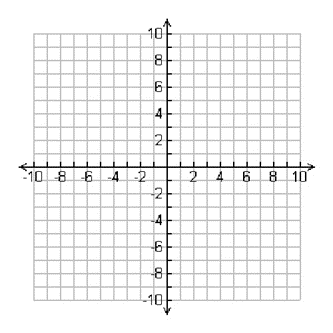
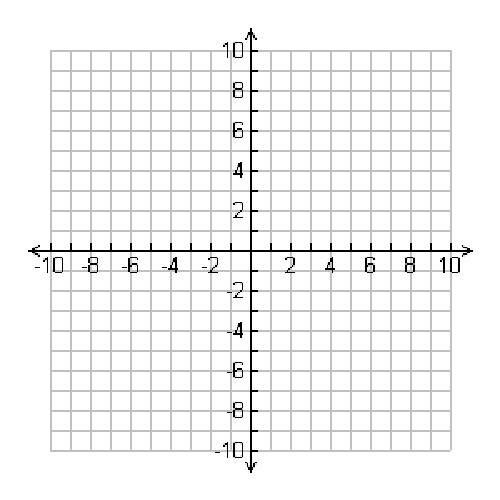
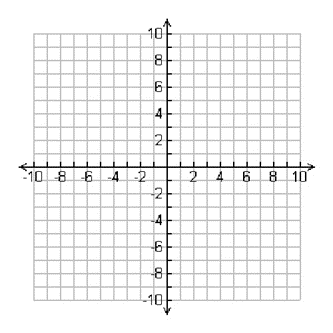


Given a function f(x), a new function g(x) = -f(x) is a vertical reflection of the function f(x), sometimes called a reflection about the x-axis.

Given a function f(x), a new function g(x) = f(-x) is a horizontal reflection of the function f(x), sometimes called a reflection about the y-axis.

Example #4: Sketch the parent graph and then the given transformation of the parent graph.

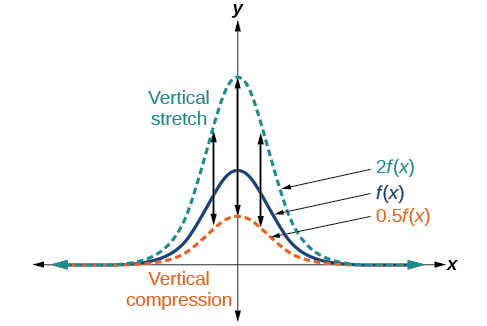
A. f(x) = - |x| B. g(x) = C. h(x) = -x

**Objective 5:** Use vertical stretching and shrinking to graph functions

Given a function f(x), a new function g(x) = *a*f(x), where *a* is a constant, is a vertical stretch or vertical compression (shrinking) of the function f(x).

* If *a* > 1, then the graph will be stretched
* If 0 < *a* < 1, then the graph will be compressed (shrunk)
* If *a* < 0, then there will be a combination of a vertical stretch or compression (shrink) with a vertical reflection

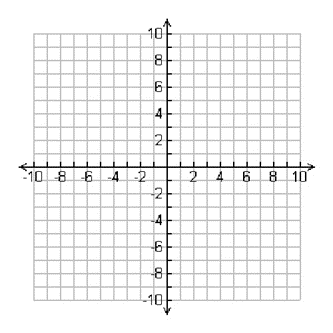
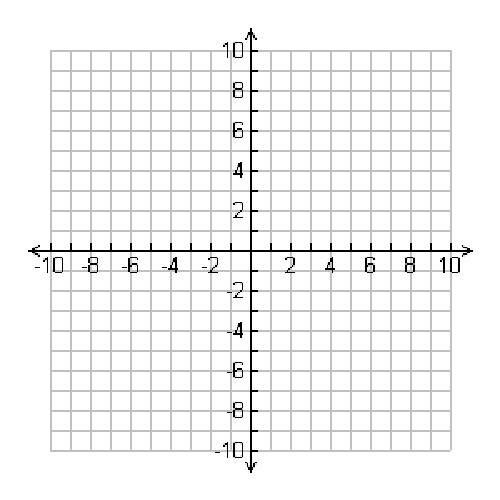
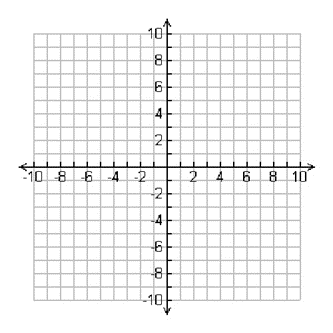


The vertical stretch goes away from the x-axis and the vertical compression (shrink) goes towards the x-axis. We multiply each of the y-coordinates by *a*.

The intercepts on the x-axis will stay in place as the graph stretches and compresses (shrinks).

Example #5: Sketch the parent graph and then the given transformation of the parent graph.

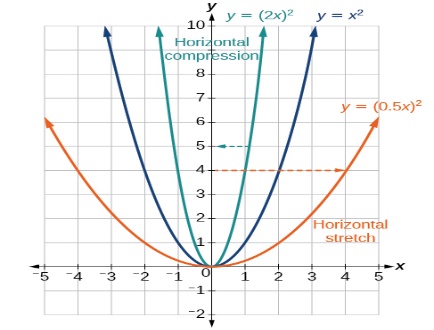
A. f(x) = 3x2 B. g(x) = C. h(x) =

**Objective 6:** Use horizontal stretching and shrinking to graph functions

Given a function f(x), a new function g(x) = f(*b*x), where *b* is a constant, is a horizontal stretch or horizontal compression (shrinking) of the function f(x).

* If *b* > 1, then the graph will be compressed (shrunk)
* If 0 < *b* < 1, then the graph will be stretched
* If *b* < 0, then there will be a combination of a horizontal stretch or compression (shrink) with a horizontal reflection

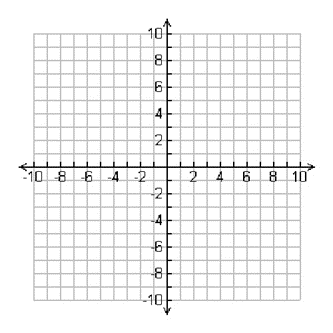
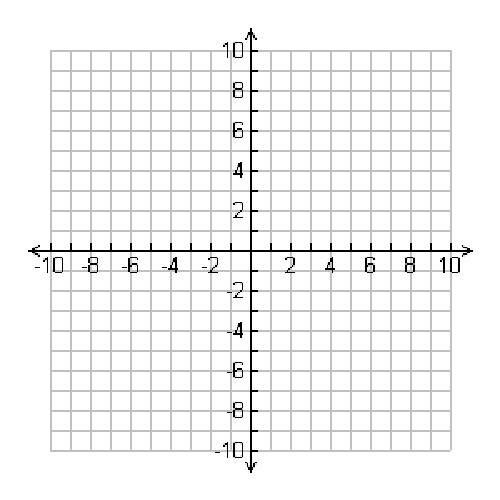
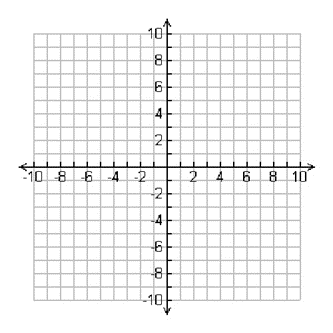


The horizontal stretch goes away from the y-axis and the vertical compression (shrink) goes towards the y-axis. We divide each of the x-coordinates by *b*.

The y-intercept will stay in place as the graph stretches and compresses (shrinks).

Example #6: Sketch the parent graph and then the given transformation of the parent graph.

A. f(x) = (3x)2 B. g(x) = C. h(x) = ( 3

**Objective 7:** Graph functions involving a sequence of transformations

We must apply the following steps when graphing a function containing more than one transformation.

1. Start with the parentheses (horizontal shift)

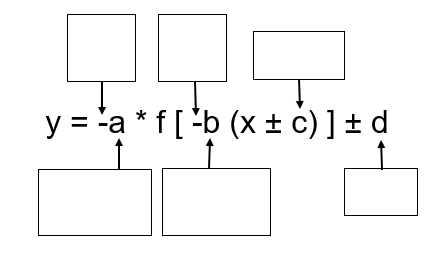
2. Deal with the multiplication or division (stretching or compressing (shrinking))

3. Deal with the negation to find the reflections

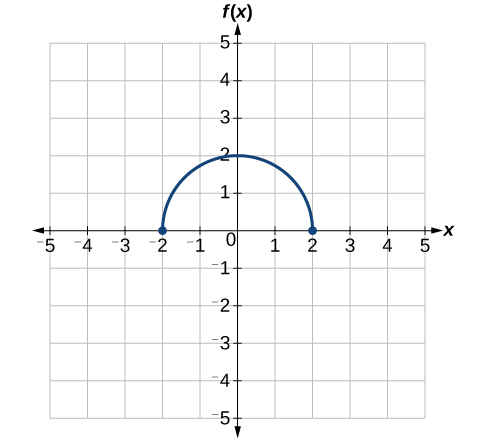
4. Deal with the adding or subtracting (vertical shift)

These steps are basically just following order of operations:

(parenthesis, multiplication/division, negation, addition/subtraction)



Example #7: Use the graph of f(x) in the figure below to sketch the graph of k(x) = ½ (x + 1) - 3. Label each transformation.



OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.

Pierce, Rod. (16 Feb 2018). "Function Transformations". Math Is Fun. Retrieved 8 Oct 2019 from http://www.mathsisfun.com/sets/function-transformations.html