Math 1111 – Logarithmic Functions

Objectives:

1. Change from logarithmic to exponential form

2. Change from exponential to logarithmic form

3. Evaluate logarithms

4. Use basic logarithmic properties

5. Graph logarithmic functions

6. Find the domain of a logarithmic function

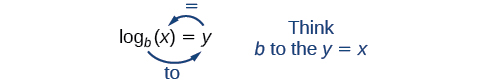
7. Use common and natural logarithms in applications

**Objective 1:** Change from logarithmic to exponential form

The definition of a logarithmic function is:

For x > 0 and b > 0, b ≠ x is equivalent to by = x. The function f(x) = logb x is the logarithmic function with base b.

Note that the base b is always positive.



Since the base of an exponential function is always positive, no power of that base can ever be negative. We can never take the logarithm of a negative number. Also, we cannot take the logarithm of zero.

We can read a logarithmic expression in a couple different ways.

1. The logarithm with base *b* of *x* is equal to *y*

2. *b* raised to the power of *y* is *x* (logs are exponents)

For example, the base 2 logarithm of 32 is 5, because 5 is the exponent we must apply to 2 to get 32. Since 25 = 32, we can write log2 32 = 5. This is read as “log base 2 of 32 is 5.”

Example #1: Write each equation in its equivalent exponential form:

A. 2 = log3 x

B. 3 = log7 x

C. log2 64 = y

**Objective 2:** Change from exponential to logarithmic form

Notice that when comparing the logarithmic function and exponential function, the input and the output are switched. This means that y = logb x and y = bx are inverse functions.



means by = x

Example #2: Write each equation in its equivalent logarithmic form:

A. 35 = x

B. 2-1 = ½

C. ey = 21

**Objective 3:** Evaluate logarithms

When given a logarithm of the form y = logb x, evaluate it mentally.

1. Rewrite the argument *x* as a power of *b*, by = x

2. Use previous knowledge of powers of *b* identify *y* by asking, “To what exponent should *b* be raised in order to get *x*?”

Example #3: Evaluate without using a calculator.

A. log10 100

B. log4 64

C. log5

D. log8 3

**Objective 4:** Use basic logarithmic properties

Common Log: A log function with a base of *10* is called a common log. It is not written but understood.

Natural log: A log function with a base of *e* is called a natural log. It is not written but understood.

Basic Properties of Logs:

1. logb b = 1 1. log 10 = 1 1. ln e = 1

2. logb 1 = 0 2. log 1 = 0 2. ln 1 = 0

3. logb bx = x 3. log 10x = x 3. ln ex = x

4. 4. 10log x = x 4. eln x = x

Example #4: Evaluate using the properties of logs.

A. log9 9

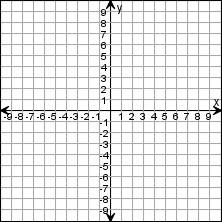
B. log8 1

C. log7 78

D.

E. ln e5

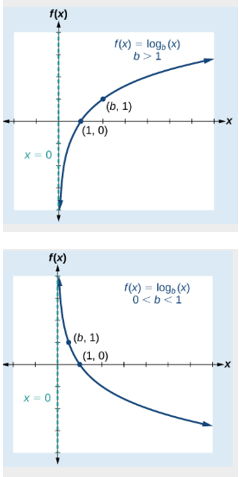
**Objective 5:** Graph logarithmic functions



Graph f(x) = 3x and g(x) log3 *x* in the same rectangular grid.

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| |  |  |  |  |  | | --- | --- | --- | --- | --- | | x | f(x) = 3x |  | x | g(x) = log3x | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   Domain: Domain:  Range: Range:  Asymptote: Asymptote: |  |

Characteristics of Logarithmic Functions of the form f(x)=logb x



1. The domain of f(x)=logb x consists of all positive real numbers; (0, ∞). The range of f(x)=logb x consists of all real numbers; (- ∞, ∞).

2. The graphs of all logarithmic functions of the form

f(x)=logb x passes through the point (1, 0) because

f(1) = 0. The x-intercept is 1. There is no y-intercept.

3. If b > 1, f(x)=logb x has a graph that goes up to the right and is an increasing function.

4. If 0 < b < 1, f(x)=logb x has a graph that goes down to the right and is a decreasing function.

5. The graph f(x)=logb x is a one-to-one function and has an inverse that is a function.

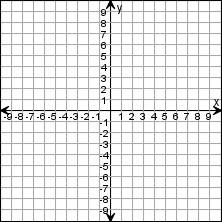
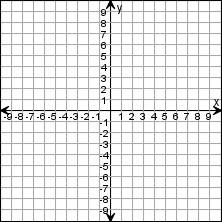
6. The graph of f(x)=logb x approaches, but does not touch, the y-axis. The y-axis, or x = 0, is a vertical asymptote.

The standard form of an exponential function is f(x) = a logb (x – h) + k. The following tables describes the transformations.

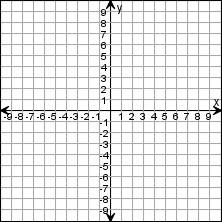
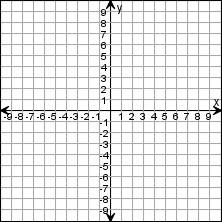
|  |  |
| --- | --- |
| Transformation | Description |
| Horizontal Shift | * If *h* > 0, then shift left *h* units * If *h* < 0, then shift right *h* units * The vertical asymptote is x = h |
| Vertical Stretching or Shrinking | Multiplying y-coordinates by *a*   * Stretches the graph if *a* > 1 * Shrinks the graph if 0 < *a* < 1 |
| Reflecting | * If *a* is negative, reflects around x-axis * If *x* is negative, reflects around the y-axis |
| Vertical Shift | * If *k* > 0, then shift upward *k* units * If *k* < 0, then shift downward *k* units |

Example #5: Graph the following exponential functions using transformations.

A. f(x) = log3 (x + 4) B. f(x) = -log3x + 2



C. f(x) = log3 (-x) D. f(x) = log3 (x – 1) - 2



**Objective 6:** Find the domain of a logarithmic function

The domain of a logarithmic function is a set of all real numbers for which the expression (x – k) is positive.

Example #4: Find the domain of the following functions. Put into interval notation.

A. f(x) = log4 (x - 5)

B. f(x) = log6 (x + 6)

C. f(x) = ln (7 - 2x)

D. f(x) = log (x2 – 3x)

**Objective 7:** Use common and natural logarithms in applications

Example #7: Solve each of the applications problems.

A. The percentage of adult height attained by a boy who is *x* years old can be modeled by

*f*(*x*) = 29 + 48.8 log (*x* + 1), where *x* represents the boy’s age (from 5 to 15) and *f*(*x*) represents the percentage of his adult height. Approximately what percentage of his adult height has a boy attained at age ten?

B. When the outside air temperature is anywhere from 72° to 96° Fahrenheit, the temperature in an enclosed vehicle climbs by 43° in the first hour. The function *f*(*x*) = 13.4 ln *x* − 11.6 models the temperature increase, *f*(*x*), in degrees Fahrenheit, after *x* minutes. Use the function to find the temperature increase, to the nearest degree, after 30 minutes.

OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.