Math 1111: Systems of Linear Equations: Two Variables

Objectives:

1. Determine if an ordered pair is a solution to a system of linear equations

2. Identify the three types of solutions for a system of equations in two variables

3. Solve systems of linear equations by substitution

4. Solve systems of linear equations by addition

5. Applications using systems of linear equations

A system of linear equations consists of two or more linear equations made up of two or more variables such that all equations in the system are considered simultaneously. To find the unique solution to a system of linear equations, we must find a numerical value for each variable in the system that will satisfy all equations in the system at the same time. For a system of equations with two unknowns, the answer will consist of an ordered pair.

**Objective 1:** Given a system of linear equations and an ordered pair, determine whether the ordered pair is a solution.

1. Substitute the ordered pair into each equation in the system.

2. Determine whether true statements result from the substitution in both equations; if so, the ordered pair is a solution.

Example #1: Determine if the ordered pair is a solution to the given systems of equations.

A. Determine if (5, 1) is a solution to the systems of equations.

x + 3y = 8

2x – 9 = y

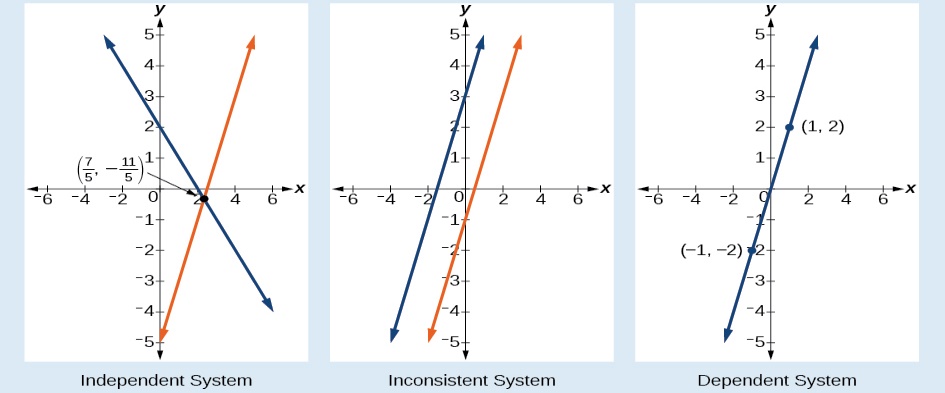
B. Determine if (8, 5) is a solution to the systems of equations.

5x - 4y = 20

2x - 3y = -1

**Objective 2:** There are three types of systems of linear equations in two variables, and three types of solutions.

* An independent system has exactly one solution pair (x, y) which is the point where the two lines intersect.
* An inconsistent systemhas no solution which means the lines are parallel and will never intersect.
* A dependent system has infinitely many solutions which means that the lines are the same, so every coordinate pair on the line is a solution to both equations.



**Objective 3:** Given a system of two equations in two variables, solve using the substitution method.

1. Solve one of the two equations for one of the variables in terms of the other.

2. Substitute the expression for this variable into the second equation, then solve for the remaining variable.

3. Substitute that solution into either of the original equations to find the value of the first variable. If possible, write the solution as an ordered pair.

4. Check the solution in both equations.

Example #2: Solve the following system of equations by substitution.

A. -x + y = -5

2x - 5y = 1

B. x = y + 3

4 = 3x – 2y

C. x = 9 – 2y

x + 2y = 13

**Objective 4:** Given a system of two equations in two variables, solve using the addition method.

1. Write both equations with the x- and y-variables on the left side of the equal sign and constants on the right.

2. Write one equation above the other, lining up corresponding variables. If one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, add the equations together, eliminating one variable. If not, use multiplication by a nonzero number so that one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, then add the equations to eliminate the variable.

3. Solve the resulting equation for the remaining variable.

4. Substitute that value into one of the original equations and solve for the second variable.

5. Check the solution by substituting the values into the other equation.

Example #3: Solve the following system of equations by substitution.

A. 3x + 5y = -11

x - 2y = 11

B. 2x + 3y = -16

5x – 10y = 30

C. x + 3y = 2

3x + 9y = 6

D.

**Objective 5:** Solve application problems using systems of linear equations.

It is important for businesses to determine when they are making a profit, otherwise they would not be able to stay in business. There are two types of functions needed to determine the profit function.

The revenue function is represented by R(x) = xp, where x = the quantity of the items produced and

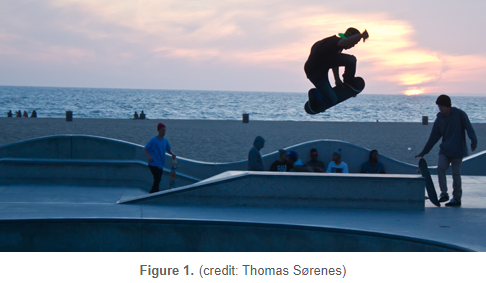
p = the price of the items to produce. This function used to calculate the amount of money that comes into the business.

The cost function is represented by C(x) = fixed costs + variable costs. The fixed costs include costs such as rent and salaries (they do not change), and variable costs, such as utilities (these change from month to month).

The break-even point is determined when the Revenue = Costs. The profit at this point is zero.

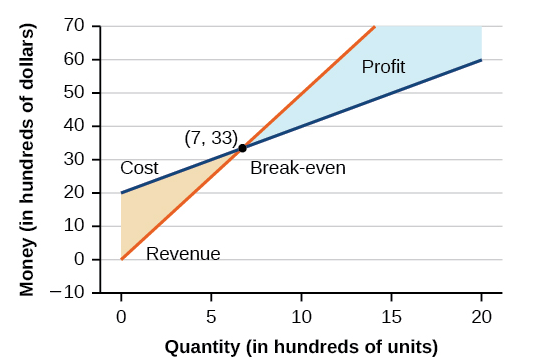
The profit function is represented by P(x) = R(x) – C(x) and when this number is positive, the business is now making money.

Consider this scenario:



A skateboard manufacturer introduces a new line of boards. The manufacturer tracks its costs, which is the amount it spends to produce the boards, and its revenue, which is the amount it earns through sales of its boards. How can the company determine if it is making a profit with its new line? How many skateboards must be produced and sold before a profit is possible?

The graph shows the Cost function and the Revenue function. The point at which the two lines intersect is at (7, 33), which means that if 700 units are produced, the cost is $3,300. The revenue is also $3,300. In other words, the company breaks even if they produce and sell 700 units. They neither make money nor lose money. The profit area shows where the company is making money and the shaded area to the left represents when the business is suffering a loss.



Example #4: Solve the application problems.

A. The start-up cost for a restaurant is $120,000, and each meal costs $10 for the restaurant to make. If each meal is then sold for $15, after how many meals does the restaurant break-even? Write the cost function, revenue function and profit function.

B. The cost of a ticket to the circus is $25.00 for children and $50.00 for adults. On a certain day, attendance at the circus is 2,000 and the total gate revenue is $70,000. How many children and how many adults bought tickets?

C. If a scientist mixed 10% saline solution with 60% saline solution to get 25 gallons of 40% saline solution, how many gallons of 10% and 60% solutions were mixed together?

D. If an investor invests $23,000 into two bonds, one that pays 4% in simple interest, and the other paying 2% simple interest, and the investor earns $710.00 annual interest, how much was invested in each account?

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