Math 1111 – Quadratic Equations

Objectives:

1. Solve quadratic equations by factoring

2. Solve quadratic equations by the square root property

3. Solve quadratic equations by completing the square

4. Solve quadratic equations by using the quadratic formula

5. Solve application problems modeled by quadratic equations

A quadratic equation is an equation containing a second-degree polynomial and can be expressed in the standard form ax2 + bx + c = 0 where a, b, and c are real numbers, and if a ≠ 0. These types of equations can be used in the fields of engineering, architecture, finance, biological science, and, of course, mathematics.

**Objective 1:** Solving a quadratic equation by using the factoring method (trial and error, grouping, or “ac” method). See notes on factoring polynomials if you need a review.

Factoring is one of the easiest methods of solving a quadratic equation. If a quadratic can be factored then it can be written as a product of two linear terms. Solving a quadratic equation depends on the zero-product rule which states: If a **∙** b = 0, then a = 0 or b = 0, where a and b are real numbers or algebraic expressions.

Solve a quadratic equation using the factoring method.

1. Factor the quadratic using any method (trial and error, grouping, or “ac” method).

2. Solve using the zero-product property by setting each factor equal to zero and solving for the variable.

3. Solve the equations that were found in #2 above.

4. Check your answers in the original equation.

Example #1: Solve the quadratic equation by factoring.

A. x2 + x – 6 = 0

B. x2 – 5x – 6 = 0

C. 2x2 + x - 3 = 0

D. 12x2 + 35x = – 8

E. x2 – 9 = 0

**Objective 2:** Solving a quadratic equation by using the square root property.

When there is no linear term, we can use the square root property.We have to isolate the *x2* term and take the square root of the number on both sides of the equal sign. We will get two answers when the square root is taken, a positive number and a negative number.

The square root property: With the *x2* term isolated, the square root property states that

If *x2* = *k*, then *x* = ±

where *k* is a nonzero real number.

Solve a quadratic equation using the square root property.

Given a quadratic equation with an *x2* term but no *x* term, use the square root property to solve it.

1. Isolate the *x2* term on one side of the equal sign.

2. Take the square root of both sides of the equation, putting a ± sign before the expression on the side opposite the squared term.

3. Simplify the numbers on the side with the ± sign.

Example #1: Solve the quadratic equation by using the square root property.

A. x2 = 8

B. 4x2 + 1 = 7

C. 3(x – 4)2 = 15

**Objective 3:** Solving a quadratic equation by completing the square.

When a quadratic equation cannot be factored or solved using the square root property, we can add or subtract terms to both sides of the equation until we have a perfect square trinomial on one side of the equation. This method is known as completing the square. Before we can begin to use the procedure to complete the square, we have to make sure that the leading coefficient is 1. If it is not, then we must divide the entire equation by that number.

Given the equation 3x2 – 12x – 15 = 0, solve by completing the square.

* Divide by 3 on both sides of the equation. x2 – 4x – 5 = 0
* Move the coefficient to the right side. x2 – 4x + \_\_\_\_\_ = 5
* Take half of the middle term and square it. x2 – 4x + (-4 \* ½)2 = 5
* Add this to the right side. x2 – 4x + 4 = 5 + 4
* Factor the left as a perfect binomial. (x – 2)2 = 9
* Use the square root property. x – 2 = ± 3
* Find both solutions. x = 3 + 2 = 5 and x = -3 + 2 = -1

Solving a quadratic equation by completing the square.

1. Make the sure the leading coefficient is 1 or divide the entire equation by that number.

2. Isolate the variables on one side of the equation and the constant on the other side.

3. Take half of the middle term (x) and square it. Add this to both sides of the equation.

4. Factor the perfect square trinomial on the left and add the terms on the right.

5. Use the square root property to solve the equation.

6. Simplify the radical and solve the two resulting equations.

7. Check the solutions.

Example #3: Solve the quadratic equation by completing the square.

A. x2 + 3x – 5 = 0

B. 2m2 + 16m + 14 = 0

C. 3x2 + 2x = 4

**Objective 4:** Solving a quadratic equation using the quadratic formula.

One method that always works when solving quadratic equations is the quadratic formula. It is very easy to make errors using this formula so **always** use parentheses when inserting a negative number.

The Quadratic Formula: Written in standard form, ax2 + bx + c = 0, any quadratic equation can be solved using the quadratic formula:

where a, b, and c are real numbers and a ≠ 0.

Solving a quadratic equation using the quadratic formula.

1. Make sure the equation is in standard form: *ax2* + *bx* + *c* = 0.

2. Make note of the values of the coefficients and constant term, *a*, *b*, and *c*.

3. Carefully substitute the values noted in step 2 into the equation. To avoid needless errors, use parentheses around each number input into the formula.

4. Calculate and solve.

Example #4: Solve the quadratic equation using the quadratic formula.

A. 9x2 + 3x – 2 = 0

B. x2 + 5x + 1 = 0

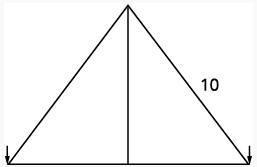
C. x2 - x + 2 = 0

**Objective 5:** Solve each of the application problems using any method above.

1. A firework is shot upwards with an initial velocity of 130 feet per second. How many seconds will it take to reach a height of 260 feet? Use the equation: h = -16*t2* + 130*t*, where *t* represents the number of seconds. Round to the nearest tenth of a second.

2. Mike wants to put 150 square feet of artificial turf in his front yard. This is the maximum area of artificial turf allowed by his homeowner’s association. He wants to have a rectangular area of turf with length one foot less than 3 times the width. Find the length and width. Round to the nearest tenth of a foot.

3. Rene is setting up a holiday light display. He wants to make a “tree” in the shape of two right triangles, as shown below, and has two 10-foot strings of lights to use for the sides. He will attach the lights to the top of a pole and to two stakes on the ground. He wants the height of the pole to be the same as the distance from the base of the pole to each stake. How tall should the pole be?



4. Charles takes his boat out and travels upstream 24 miles and back again. The speed of the boat is 11 mph and it takes a total of 5.5 hours for his trip. Find the rate of the current.

OpenStax, Intermediate Algebra. OpenStax CNX. Jun 11, 2019 http://cnx.org/contents/02776133-d49d-49cb-bfaa-67c7f61b25a1@8.1.

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