Math 1111: Distance and Midpoint Formulas; Circles

Objectives:

1. Find the distance between two points.

2. Find the midpoint of a line segment

3. Standard equation form of a circle

a. Write the standard form of a circle’s equation

b. Give the center and radius of a circle whose equation is in standard form

4. Convert the general form of a circle’s equation to standard form

**Objective 1:** Find the distance between two points

When you want to find the distance between two points in the coordinate plane, you can use the distance formula.

We can derive the distance formula by using the Pythagorean Theorem and substituting | *x2 – x1* | for *a*, | *y2 – y1* | for *b* and *d* for *c*. Since we want the numbers to be positive, we are using absolute value.

Pythagorean Theorem: a2 + b2 = c2

Substitution: ( | *x2 – x1* | )2 + ( | *y2 – y1* | )2 = d2

Drop the absolute value since

squaring makes the expression positive: ( *x2 – x1* )2 + ( *y2 – y1* )2 = d2

Use the square root property:

Only use the “+” since distance can only

be positive.

Therefore, given two points (x1, y1) and (x2, y2), the distance between them is given by,

**Example #1:** Find the distance between the two points. Give the answer in radical form and then to one decimal place, if needed.

A. (2, 6) and (-2, 5)

B. (-5, -3) and (1, -7)

C. (-2, -5) and (-3, -4)

**Objective 2**: Find the midpoint of a line segment.

It is often useful to be able to find the midpoint of a segment. For example, if you have the endpoints of the diameter of a circle, you may want to find the center of the circle which is the midpoint of the diameter. To find the midpoint of a line segment, we find the average of the x-coordinates and the average of the y-coordinates of the endpoints.

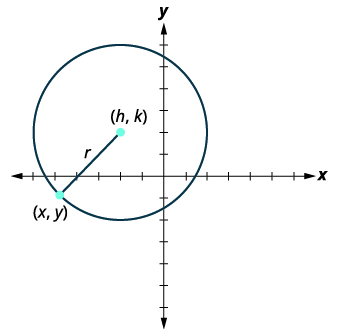
Given two points (x1, y1) and (x2, y2), the midpoint M is given by,

**Example #2:** Find the midpoint given the endpoints of the line segment.

A. (-3, -5) and (9, 7)

B. (-2, -5) and (6, -1)

C. Given the midpoint is (1, -1) and an endpoint is (7, 2) can you find the other endpoint?

**Objective 3a and b:** Write the standard form of a circle’s equation

A circle is all points in a plane that are a fixed distance from a given point in the plane. The given point is called the center (*h, k*), and the fixed distance is called the radius, *r*, of the circle.

We can derive the equation for a circle by using the distance formula and substituting in (*h, k*) for (*x, y*) and *r* for *d*.

Distance Formula:

Substitution:

Square both sides: r2 = (x – h)2 + (y – k)2

Therefore, the standard form of the equation of a circle with center (*h, k*), and radius*, r*, is

r2 = (x – h)2 + (y – k)2

**Example #3a:** Write the standard form of the equation of the circle given the information.

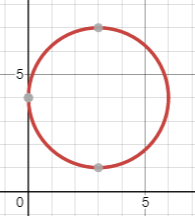
A. Radius 2 and center at (-1, 3)

B. Radius 5 and center at (-2, -4)

C. Diameter 16 and center at (3, 1)

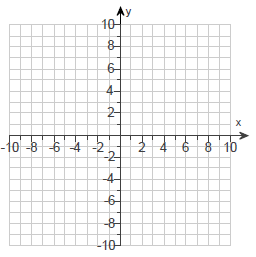
D. The diameter has endpoints of (-2, 1) and (4, 3)

E. Center (-1, 4) and contains (2, -2)



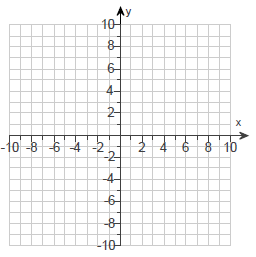
F.

**Example #3b:** Given the standard equation of a circle, find the radius and center. Graph the equation and determine the domain and range.

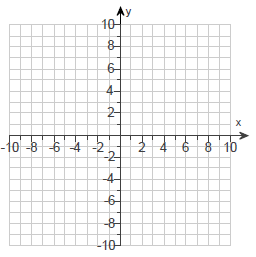


A. (x – 4)2 + (y + 5)2 = 36

B. (x + 1)2 + y2 = 16



C. x2 + (y – 2)2 = 4



**Objective 4:** Convert the general form of a circle’s equation to standard form

The general form of the equation of a circle is x2 + y2 + ax + by + c = 0. In order to find the center and radius to graph the circle, we can complete the square to get the equation into the standard form.

Given the equation x2 + y2 – 4x – 6y + 4 = 0, find the center and the radius.

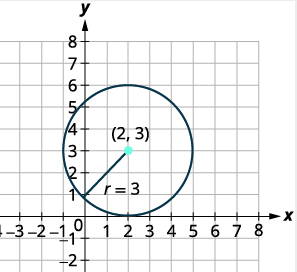
Group the x and y terms

move the constant to the right: x2 – 4x + y2 – 6y = -4

Complete the square: x2 – 4x + 4 + y2 – 6y + 9 = -4 + 4 + 9

Rewrite as binomial squares: (x – 2)2 + (y – 3)2 = 9

Now you have the equation written in standard form and can determine that the center is (2, 3) and the radius is 3.



**Example #4:** Write the equation of the circle in standard form by completing the square. Determine the center and the radius.

A. x2 + y2 – 6x – 7y + 9 = 0

B. x2 + y2 + 6x – 2y + 1 = 0

C. x2 + y2 + 8y = 0

D. x2 + y2 – 2x - 3 = 0

E. 2x2 + 2y2 - 8x + 6y - 7 = 0

OpenStax, Intermediate Algebra. OpenStax CNX. Jun 11, 2019 http://cnx.org/contents/02776133-d49d-49cb-bfaa-67c7f61b25a1@8.1.