Math 1111 – Inverse Functions

Objectives:

1. Verify inverse functions

2. Find the inverse of a function

3. Use the horizontal line test to determine if a function has an inverse function

4. Use the graph of a one-to-one function to graph its inverse function on the function on the same axes

**Objective 1:** Verify inverse functions

An inverse function is a function for which the input of the original function becomes the output of the inverse function, and the output of the original function becomes the input of the inverse function.

Given a function f(x), we represent its inverse as f-1(x), read as “f inverse of f”. The raised -1 is part of the notation. It is not an exponent and does not imply a power of -1. In other words, f-1(x) ≠.

Given a function f(x), we can verify whether some other function g(x) is the inverse of f(x) by checking whether either g(f(x)) = x or f(g(x)) = x is true.

Definition of inverse functions: Suppose *f* and *g* are two functions such that

1. (*g ∘ f*) (x) = *x* for all *x* in the domain of *f* and

2. (*f ∘ g*) (x) = *x* for all *x* in the domain of *g*

then *f* and *g* are inverses of each other and the functions *f* and *g* are said to be invertible.

We now formalize the concept that inverse function exchange inputs and outputs.

Properties of Inverse Functions (Theorem 5.2): Suppose *f* and *g* are inverse functions.

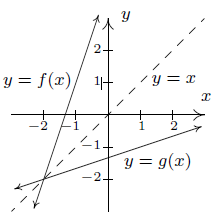
1. The range of *f* is the domain of *g* and the domain of *f* is the range of *g*

2. f(a) = b if and only if g(b) = a

3. (a, b) is on the graph of *f* if an only if (b, a) is on the graph of *g*

The third property in Theorem 5.2 implies that inverse functions are symmetric about the line

y = x. For example, we plot the inverse functions f(x) = 3x + 4 and g(x) = below.



Example #1: Verify if each of the functions are inverses of each other.

A. f(x) = 4x and g(x) =

B. f(x) = and g(x) *=*

**Objective 2:** Find the inverse of a function

Not every function has an inverse function. A function whose inverse is also a function is called a one-to-one function.

In order to find the inverse of a function we follow a simple four step process:

1. Replace f(x) with y

2. Switch *x* and *y*

3. Solve for the *y*

4. Replace *y* with f-1(x)

Example #2: Given that the following functions are one-to-one functions, find their inverses.

A. f(x) = 3x + 1

B. f(x) = , x ≥ 0

C. f(x) =

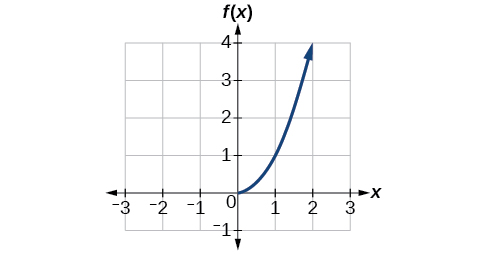
D. f(x) = x3 - 1

**Objective 3:** Use the horizontal line test to determine if a function has an inverse function

A function is called one-to-one if no two values of x produce the same y. A one-to-one function passes the horizontal line test. If any horizontal line crosses the graph of a function more than once, that means that y-values repeat and the function is not one-to-one. However, if a function is restricted to a certain domain so that it passes the horizontal line test, then in that restricted domain, it can have an inverse.

If no horizontal line crosses the graph of the function more than once, then no y-values repeat and the function is one-to-one.

Let us look at the quadratic function f(x) = x2. It must be restricted to the domain [0,∞), for this function to be considered one-to-one. See the figure below.

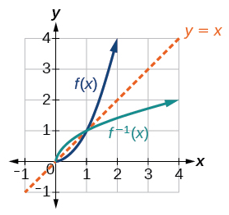


Quadratic function with domain restricted to [0, ∞).

We already know that the inverse of the restricted quadratic function is the square root function, this is,

f-1(x) = . What happens if we graph both f and f-1 on the same set of axes?

We notice a distinct relationship: The graph of f-1(x) reflected about the diagonal line y = x, which we will call the identity line. See the figure below.



Square and square-root functions on the non-negative domain

**Objective 4:** Use the graph of a one-to-one function to graph its inverse function on the function on the same axes

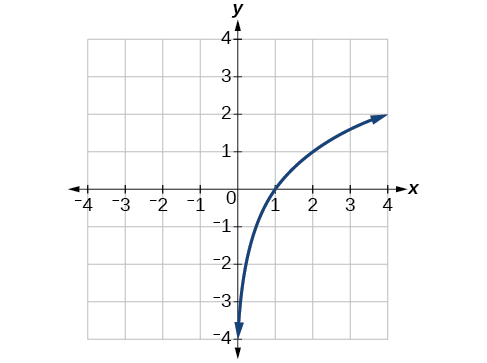
The graph of an inverse function can be obtained by using the reflection over the line y = x. You can also reverse the order of the coordinate points.

For example, if the function has coordinate pairs of (-2, -1), (-1, 2), (0, 5), (1, 8) and (2, 11) the inverse of the function has coordinate pairs of (-1, -2), (2, -1), (5, 0), (8, 1) and (11, 2).

Example #4: Practice with sketching inverses.

A. Given the graph of f(x) in the B. Given the equation y = 3x3 – 4, plot some points. figure, sketch the graph of f-1(x) then sketch the graph and its inverse.

 on the same axes.



OpenStax College Algebra, College Algebra. OpenStax CNX. Aug 2, 2019 http://cnx.org/contents/9b08c294-057f-4201-9f48-5d6ad992740d@11.1.

OpenStax CAT, Algebra and Trigonometry. OpenStax CNX. Aug 8, 2019 <http://cnx.org/contents/13ac107a-f15f->49d2-97e8-60ab2e3b519c@17.5.