4.7 Optimization Problems

Where possible, we will follow this 5-step procedure for solving applied max-min problems.

Step 1. Draw an appropriate figure and label the quantities relevant to the problem.

Step 2. Find a formula for the quantity to be maximized or minimized.
This is called the **primary equation**.

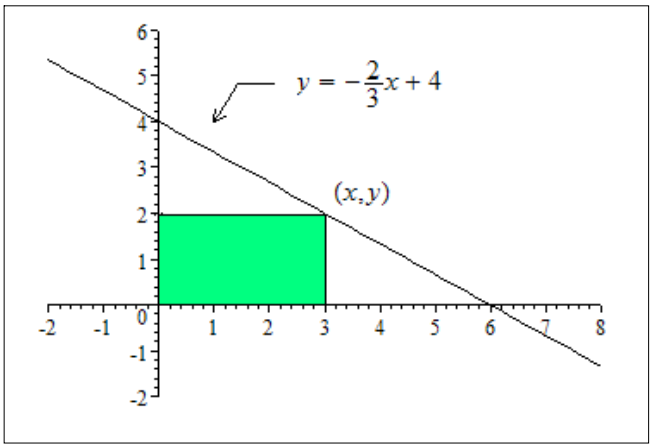
Step 3. Find an equation which relates the independent variables in the primary equation.
This is called a secondary equation. Using the secondary equation to eliminate variables from the primary equation until the the quantity to be maximized or minimized is expressed as a function of a single variable.

Step 4. Find the interval of possible values for this variable from the physical restrictions of the problem.

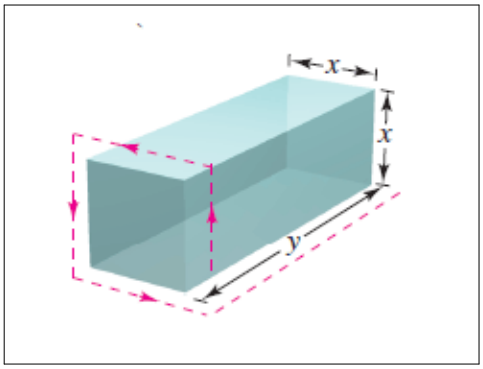
Step 5. If applicable, use the techniques of the preceding sections to obtain the maximum or minimum.

Example 1 Determine the dimensions of a rectangular solid with a square base and no top if its volume is to be maximized and if its surface area is to be 500 square inches.

Example 2 A rectangle is bounded by the x –and y –axes and the graph of $y = -\frac{2}{3}x + 4$ (see figure). What length (x) and width (y) should the rectangle have so that its area is a maximum?



Example 3 A rectangular package to be sent by a postal service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches (see figure). Find the dimensions of the package of maximum volume that can be sent. (Assume the cross section is square.)



Example 4 An offshore oil well is 4 kilometers off the coast. The refinery is 8 kilometers down the coast (see figure). Laying pipe in the ocean is twice as expensive as on land. What path should the pipe follow in order to minimize the cost?