5.1 Antiderivatives and Indefinite Integration

DEFINITION. A function F is called an **antiderivative** of a function f on a particular interval I if $F'(x) = f(x)$ for all x in the interval.

Example 1 Find three different antiderivatives of $f(x) = 3x^2$.

THEOREM If F is any antiderivative of f on an interval I, then for any constant C, the function $G(x) = F(x) + C$ is also an antiderivative of f on that interval. Moreover, each antiderivative of f on the interval I can be expressed in the form $F(x) + C$ by choosing the constant C appropriately.

If $\frac{d}{dx}[F(x)] = f(x)$, all antiderivatives of $f(x)$ are of the form $F(x) + C$.
The process of finding antiderivatives is called **antidifferentiation** or **integration**.

The operation of integration is denoted as follows:

$$\int f(x)dx = F(x) + C$$

For example, the antiderivatives of $f(x) = 4x^3$ are functions of the form $F(x) = x^4 + C$.
So, $\int 4x^3 dx = x^4 + C$.

The symbol $\int$ is called the **integral sign**. The expression $f(x)$ is called the **integrand**.
The symbol dx indicates that x is the **variable of integration**.
That is, the integration is taking place **with respect to** the variable x.
The constant C is called the **constant of integration**.
The statements $\frac{d}{dx}[F(x)] = f(x)$ and $\int f(x)dx = F(x) + C$ are equivalent statements.

Example 2 Write an equivalent integration formula for each derivative formula given below.
The first one is done for you.

$\frac{d}{dx}[4x] = 4$	$\int 4dx = 4x + C$
$\frac{d}{dx}\left[\frac{1}{5}x^5\right] = x^4$	
$\frac{d}{dx}[6x^{2/3}] = 4x^{-1/3}$	
$\frac{d}{dx}[\sin x] = \cos x$	
$\frac{d}{dx}[e^{3x}] = 3e^{3x}$	

Example 3 Evaluate each integral.

a. $\int x^5 dx =$

b. $\int \frac{1}{x^3} dx =$

c. $\int x^{2/3} dx =$

Properties of the Indefinite Integral

THEOREM.

(a) A constant factor can be moved through the integral sign.

$$\int cf(x)dx = c\int f(x)dx$$

(b) An antiderivative of a sum is the sum of the antiderivatives; that is,

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

(c) An antiderivative of a difference is the difference of the antiderivatives; that is,

$$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$$

Example 4 Evaluate each integral.

a. $\int 3\sin x dx =$

b. $\int (3x^5 - 2x^4)dx =$

Example 5 Evaluate each integral.

a. $\int \frac{3t + 7t^3}{t^2} dt =$

b. $\int \frac{1}{1 + \cos x} dx =$

Initial Conditions and Antiderivatives

Example 6

a. Find the general solution to the equation $F'(x) = 3x^2 - 4$.

b. Find the particular solution to this equation that satisfies the initial condition $F(1) = 3$.

Integration from the Viewpoint of Differential Equations

Example 7 A rock is thrown upward from an initial height of 40 m with an initial velocity of 20 m/s. Using -9.8 m/s^2 as the acceleration due to gravity near the surface of the Earth, find an expression for the height $s(t)$ of the rock in meters as a function of the time t in seconds.