2.2 Finding Limits Graphically and Numerically

LIMITS (AN INFORMAL VIEW). If the values of $f(x)$ can be made as close as we like to L by making x sufficiently close to a (but not equal to a), then we write

$$\lim_{x\to a} f(x) = L$$

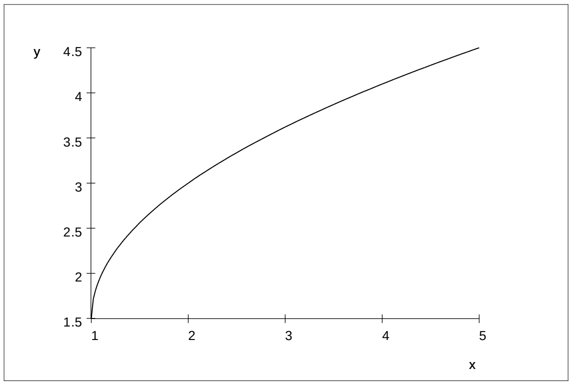
which is read "the limit of $f(x)$ as x approaches a is L."

Example 1 Make a conjecture about the value of the limit $\lim_{x\to 2} \frac{3x-6}{\sqrt{4x-4}-2}$.

The function $f(x) = \frac{3x-6}{\sqrt{4x-4}-2}$ is undefined at $x = 2$, but the values of $f(x)$ appear to be approaching a particular number as x approaches 2.

This number is "the **limit** of $f(x)$ as x approaches 2." Find it.

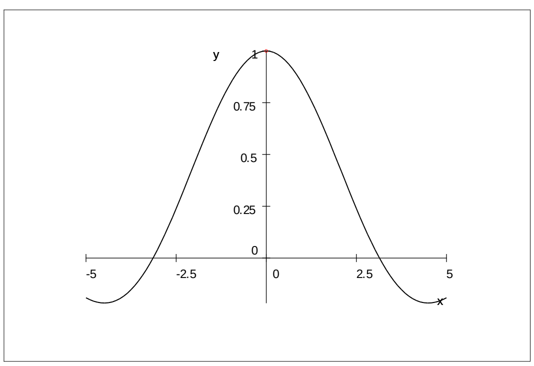
$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{3x-6}{\sqrt{4x-4}-2} =$$



x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	2.923	2.9925	2.9993	?	3.0008	3.0075	3.0732

Example 2 Make a conjecture about the value of the limit $\lim_{x\to 0} \frac{\sin x}{x}$.

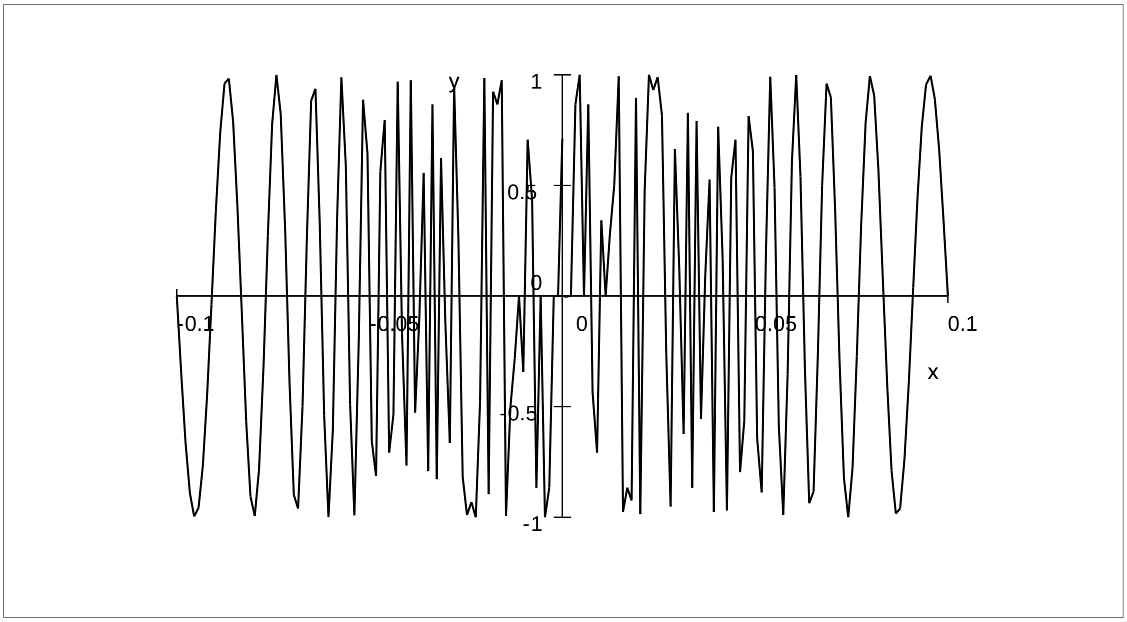
Let $f(x) = \frac{\sin x}{x}$. Find $\lim_{x\to 0} f(x)$. $\qquad \lim_{x\to 0} \frac{\sin x}{x} =$



x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	.9983341	.9999833	0.9999998	?	0.9999998	0.999833	0.9983341

Example 3 Make a conjecture about the value of the limit $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$.

$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) =$



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x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0	0	0	?	0	0	0

x	$-\frac{2}{3}$	$-\frac{2}{5}$	$-\frac{2}{7}$	$-\frac{2}{9}$	0	$\frac{2}{9}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{2}{3}$
$f(x)$	1	-1	1	-1	?	1	-1	1	-1

A Formal Definition of Limit

Intuitively speaking, the limit of a function f at a number c in its domain is the number L that the values of $f(x)$ are approaching as the values of x are approaching c.

Let $f(x)$ be defined for all x in some open interval containing the number c,
with the possible exception that $f(x)$ need not be defined at c.
The statement $\lim_{x \to c} f(x) = L$ means that for each number $\varepsilon > 0$ there exists
a number $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$