4.6 A Summary of Curve Sketching

Example 1 Let $s(t) = t^3 - 5t^2 - 8t + 40$.

(a) Find the zeros of $s(t)$.

(b) Sketch a sign graph for $s(t)$.

(c) Find $s'(t)$ and find all first-order critical numbers.

(d) Sketch a sign graph for $s'(t)$. Indicate the intervals where $s(t)$ is increasing and where it is decreasing. Use the first derivative test to identify the location of any relative extrema.

(e) Find $s''(t)$ and find all second-order critical numbers. Indicate the intervals where $s(t)$ is concave up and where it is concave down. Find any inflection points of the graph of $s(t)$.

Rectilinear motion is the motion of a particle which is allowed
to move in either direction along a coordinate line. The coordinate of the particle at time t will be denoted be $s(t)$. The function $s(t)$ is called the **position function** of the particle. When we graph $s(t)$ along the vertical axis and t along the horizontal axis, we will call this the **position versus time curve**.

Example 1 Consider the following position versus time graph of a particle moving. Discuss the motion of the particle during the indicated time interval.
Mr. Math will now attempt to draw a position versus time curve and then
use it to discuss the motion of the particle. Please be patient. He will break the domain of the position function into several subintervals and in each one indicate whether the particle is on the positive or negative side of the origin, whether the particle is headed in the positive or negative direction, and whether the particle is speeding up or slowing down.

DEFINITION. If $s(t)$ is the position function of a particle moving on a coordinate line, then the **instantaneous velocity** of the particle at time t is defined by
$$v(t) = s'(t) = \frac{ds}{dt}$$

If $v(t) > 0$ then $s(t)$ is *increasing* and the particle is *moving in the positive direction*.
If $v(t) < 0$ then $s(t)$ is *decreasing* and the particle is *moving in the negative direction*.

If we are interested in how fast a particle is moving without regard to the direction of the motion, we a concerned with its **speed** rather than its velocity. The speed of a particle is defined to be the absolute value of its velocity.

$$\begin{bmatrix} \text{Instantaneous} \\ \text{speed} \end{bmatrix} = |v(t)| = \left|\frac{ds}{dt}\right| \text{ or } |s'(t)|$$

DEFINITION. If $s(t)$ is the position function of a particle moving on a coordinate line, then the instantaneous acceleration of the particle at time t is defined by

$$a(t) = v'(t) = \frac{dv}{dt} \quad \text{or} \quad a(t) = s''(t) = \frac{d^2s}{dt^2}$$

Interpreting the Sign of Acceleration

There are four significant cases to consider.

(1) $v(t) > 0$ and $a(t) > 0$. If the velocity of a particle is positive and increasing, then its absolute value $|v(t)|$ is increasing, so the particle is *speeding up*.

(2) $v(t) > 0$ and $a(t) < 0$. If the velocity of a particle is positive but decreasing, then its absolute value $|v(t)|$ is decreasing, so the particle is *slowing down*.

(3) $v(t) < 0$ and $a(t) > 0$. If the velocity of a particle is negative but increasing, then its absolute value $|v(t)|$ is decreasing, so the particle is *slowing down*.

(4) $v(t) < 0$ and $a(t) < 0$. If the velocity of a particle is negative and decreasing, then its absolute value $|v(t)|$ is increasing, so the particle is *speeding up*.

Interpreting the Sign of Acceleration. A particle in rectilinear motion is *speeding up* when its velocity and acceleration have the *same sign* and *slowing down* when they have *opposite signs*.

Analyzing the Position Versus Time Curve

Based on the signs of the position versus time function, and the signs of its first and second derivatives, we will classify the activity of the particle as being either on the positive or negative side of the origin, moving in the positive direction or the negative direction, and speeding up or slowing down.

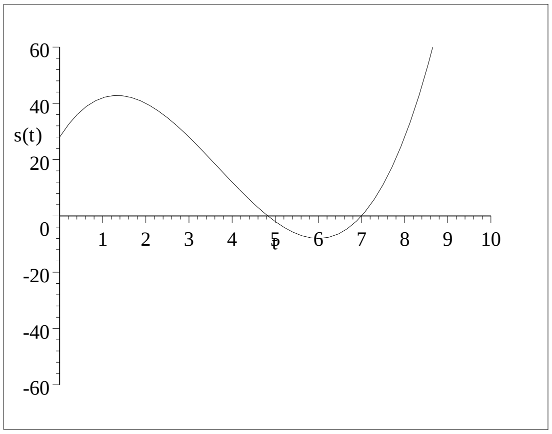
Graph	**Location** **Sign of s(t)**	**Direction** **Sign of v(t)**	**Sign of a(t)**	**Speeding up or Slowing down** Are the signs of $v(t)$ and $a(t)$ the same or are they different?

Example 2 Suppose that the position function of a particle moving on a coordinate line is given by $s(t) = t^3 - 11t^2 + 24t + 28$. Analyze the motion of the particle for $t \geq 0$.
The location, direction, or acceleration of the particle can change at a zero of $s(t)$, $v(t)$, or $a(t)$.
So the intervals to be considered are those formed by zeros of these three functions.

$s(t) = t^3 - 11t^2 + 24t + 28$
$s(t) = (t-7)(t^2 - 4t - 4)$
The zeros of $s(t)$ are $2 + 2\sqrt{2} \approx 4.8$, 7, and $2 - 2\sqrt{2}$. Here, we will exclude $2 - 2\sqrt{2}$ from consideration because it is less than 0.

$v(t) = 3t^2 - 22t + 24$
$v(t) = (3t-4)(t-6)$
The positive zeros of $v(t)$ are $t = \frac{4}{3}$ and $t = 6$.

$a(t) = 6t - 22$
The only zero of $a(t)$ is $t = \frac{11}{3}$ and it is positive.



If we list all nonnegative zeros of $s(t)$, $v(t)$, and $a(t)$ in increasing order, they give us the intervals of the number line that we need to consider.
$\frac{4}{3}, \frac{11}{3}, 2 + 2\sqrt{2}, 6, 7$

Graph	Location Sign of s(t)	Direction Sign of v(t)	Sign of a(t)	**Speeding up or Slowing down** Are the signs of $v(t)$ and $a(t)$ the same or are they different?
$\left(0, \frac{4}{3}\right)$				
$\left(\frac{4}{3}, \frac{11}{3}\right)$				
$\left(\frac{11}{3}, 2 + 2\sqrt{2}\right)$				
$(2 + 2\sqrt{2}, 6)$				
$(6, 7)$				
$(7, \infty)$				